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A THERMAL TESTING FACILITY FOR THE AERONAUTICS

DEPARTMENT AT USNPS

by

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Submitted in partial fulfillment

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ABSTRACT

An engineering study was conducted of a testing
facility for the Aeronautics Department at USNPS designed
to test for thermal effects on aircraft structures. Types
of equipment and testing methods are described. A recom-
mended facility employing radiant quartz tube lamps as a
heat source and mechanical loading is described in detail.
Cost breakdowns of this and alternate facilities are com-
pared. Additional recommended equipment and typical uses
are discussed.

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TABLE OF SYMBOLS

a = absorptivity factor

c = volumetric specific heat

E = modulus of elasticity

h = convective heat transfer coefficient

k = conductivity

L = length

M = Mach number

q = heat flow per unit area

T = temperature

Δ T = temperature increase

t = time

λ = constant of radiation

δ = deflection

σ = stress

α = coefficient of thermal expansion

μ = Poisson's ratio

SUBSCRIPTS

aw = adiabadic wall

m = model

o = free stream

p = prototype

s = surface

st = stagnation

I. INTRODUCTION

Man has been observing thermal effects on airborne objects since the first meteor was seen streaking across the night sky. Detailed study in the field, however, has been confined to the twentieth century. Since the relatively recent development of supersonic and hypersonic vehicles, the study of the effects of aerodynamic heating on aircraft structures has become increasingly important. A survey of the literature will show that considerable research is still to be done in this field.

No graduate of a program of study in aeronautics can consider his education complete without having studied the effects of thermal stresses on aircraft structures and performance. The program of study at the United States Postgraduate School in Monterey, California now includes the study of thermal effects; however, additional laboratory facilities are necessary with which to demonstrate classic examples. In addition the facilities should be such that small scale research work may be pursued in conjunction with thesis and dissertation work by both students and faculty.

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This paper will be concerned with an investigation of what experimental testing should be done, what laboratory equipment should be procured, how much it is likely to cost, how it shall be installed in conjunction with existing equipment, and finally some typical problems which may be investigated with it.

Research for this paper was done at the U. S. Naval Postgraduate School under the guidance of Professor C. Kahr whose assistance is gratefully appreciated.

II. HIGH TEMPERATURE CAUSES AND EFFECTS

Aerodynamic Heating

The primary source of heat in a supersonic vehicle
is aerodynamic heating. This is caused by the large
amounts of energy which are imparted, essentially by
compressive and viscous effects, to relatively small air
masses adjacent to the aircraft surfaces. Most of this
energy is retained at the surface in the form of heat.
The temperature in the boundary layer surrounding the
airplane, therefore, is much higher than the surrounding
stream, and heat is conducted into the aircraft.

To get some idea of the magnitude of temperatures
encountered, consider a body moving at a velocity V in
a gas at rest. At the stagnation point of the fluid
flow field, the total kinetic energy of the relative
velocity is transformed into heat energy. If it is as-
sumed that no heat is lost, the stagnation temperature
can be calculated from the following formula:

$$T_{st} = T_O \ (1+0.2M^2)$$

where T_{st} = stagnation temperature (^{O}R)

T_O = free stream temperature (^{O}R)

M = Mach number of the flow.

Theoretically this stagnation temperature exists at the nose of the aircraft and leading edges of flight surfaces. A laminar boundary layer develops at these leading edges and changes into a turbulent boundary layer at some point downstream. The temperature at the interface of the boundary layer, called the adiabadic wall temperature, can be calculated by modifying the above formula as follows:

$$T_{aw} = T_o \ (1+0.2RM^2)$$

where symbols are as before and R is the recovery factor which may be thought of as a convective efficiency factor. This recovery factor is mainly dependent upon whether the boundary layer is laminar or turbulent, being slightly higher for turbulent flow.

Fig. 1 shows a plot of temperature versus Mach number at an arbitrary altitude of 50,000 ft. The dashed line represents stagnation temperature which is not attainable in practice due to conduction and radiation losses to the atmosphere. Losses in the boundary layer are accounted for by the recovery factor which is usually between 0.8 and 0.9. A fair representation of the surface temperature at a stagnation point of an aircraft flying at 50,000 ft. will therefore lie between the 0.8 and 0.9 recovery factor lines. (1)

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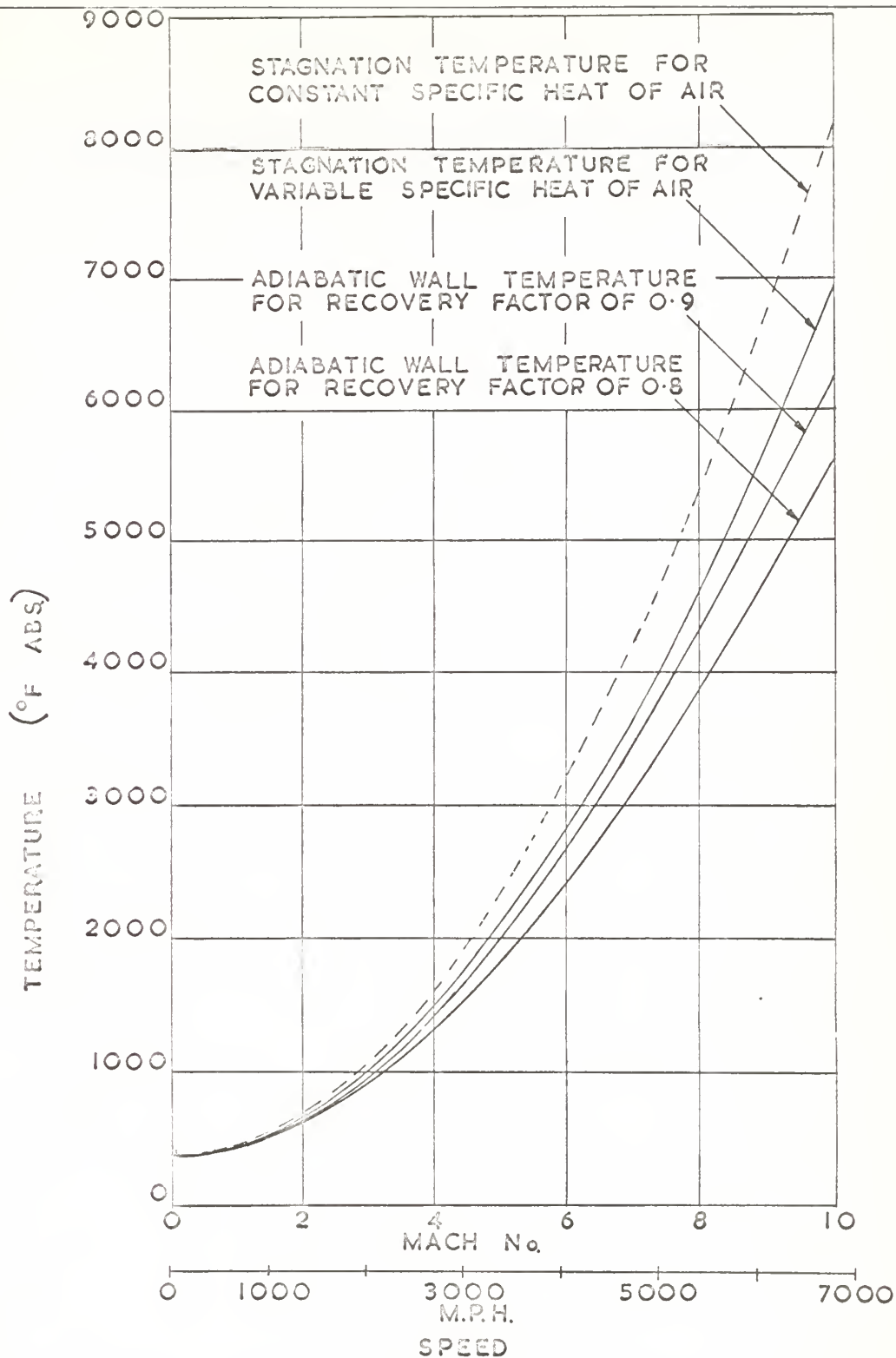
FIG. I STAGNATION AND ADIABATIC WALL TEMPERATURES FOR FLIGHT OF CONSTANT SPEED IN AIR AT 50,000 FEET (REPRODUCED FROM REF. 1)

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Newton's law of convective heat transfer can be
expressed as follows:

$$q = h \ (T_{aw} - T_s)$$

where q = heat flow between BL interface and surface

(BTU/ft^2)

h = the heat transfer coefficient $(BTU/hr/ft^2/^\circ F)$.

A major difficulty in using this equation is the
fact that h depends on many variables. Primarily, these
are the pressure distribution on the wing and the form
of the boundary layer - whether laminar or turbulent.
For example, the heat transfer in a turbulent boundary
layer is so much greater than in a laminar boundary layer
that the surface temperature may, under certain condi-
tions, even exceed the stagnation temperature. (1)

Fig. 2 shows the combined effects of all heat sources
on the chordwise temperature distribution of a wedge shaped
wing at 50,000 ft. The high temperature at the leading
and trailing edges is due to the fact that these areas are
at or near stagnation conditions. The dip in the center
area is due to a Prandtl-Meyer expansion at a surface dis-
continuity at this point.

The previous considerations were for the case in
which the assumption was made that the total heat transfer

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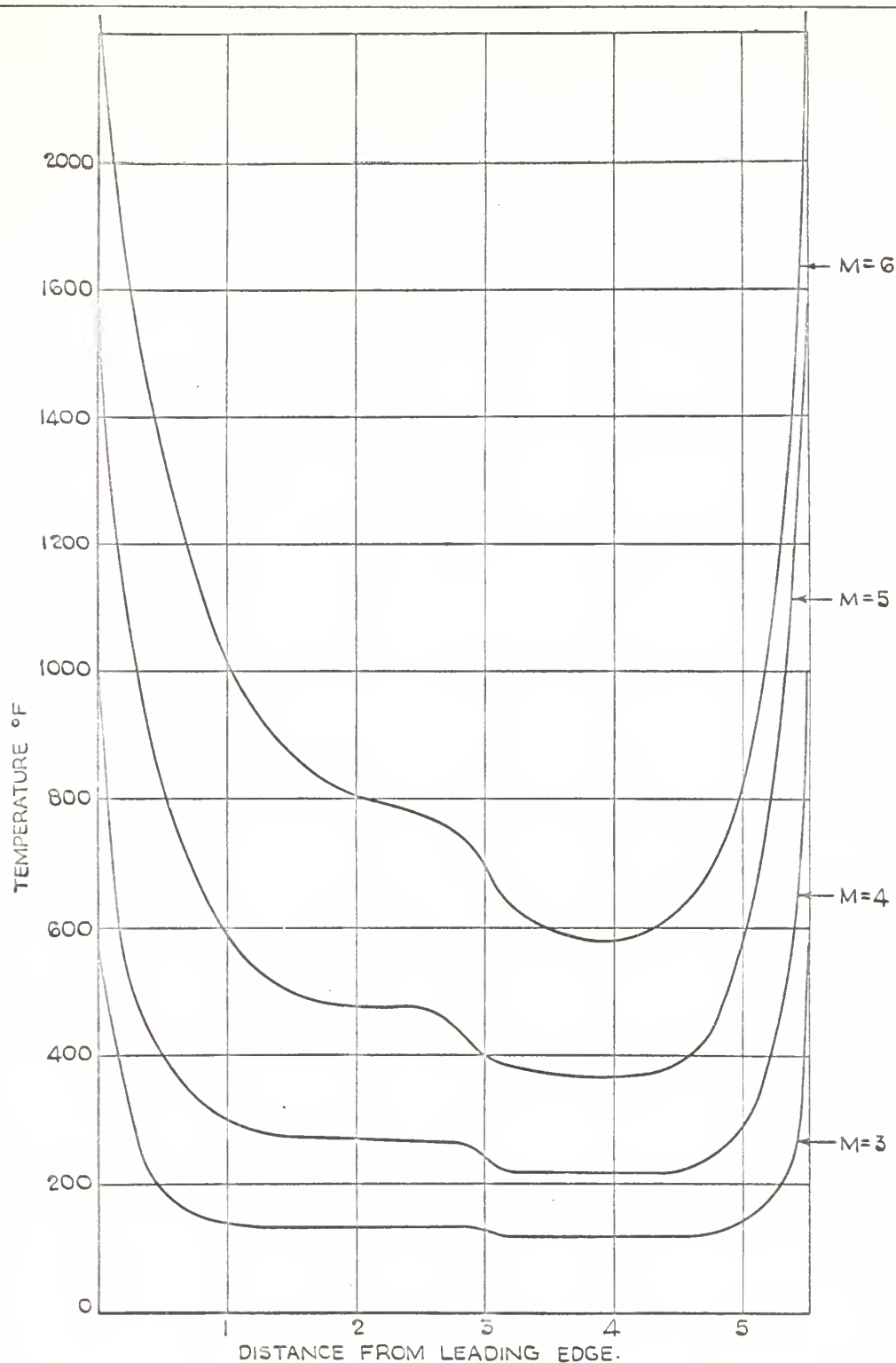
FIG. 2 CHORDWISE VARIATION OF SURFACE
TEMPERATURE WITH MACH NUMBER FOR
WEDGE SHAPED WING FOR ACCELERATION
OF 1g AT 50,000 FT.

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was due to aerodynamic heating. To completely define the heating problem, radiation must be considered, since all bodies, whether moving or stationary, continuously emit and absorb radiation. A better formula for heat flow into the structure therefore is:

$$q = h(T_{aw} - T_s) - \lambda T_s^4 + f(S)$$

where λT_s^4 = the Stefen-Boltzman radiation loss to free space $(BTU/hr/ft^2)$

$f(S)$ = solar radiation gain $\simeq$ 400 $BTU/hr/ft^2$ (1).

Other Sources of Heat

Temperature gradients due to aerodynamic heating are not the only type encountered. In addition to a chord-wise temperature gradient previously described, there may be thermal gradients within the wing perpendicular to the surface. These gradients may be caused by cooling of fuel within the wings or may be due to conduction into ribs and spars. Also, there may be hot areas near engine exhaust outlets.

High Temperature Effects on Structures

The primary effect of high temperatures on structural members is to reduce the value of the elastic limit, the yield point, the failing stress, and the modulus of elas-

ticity of the materials of construction. Fig. 3 shows how temperature effects the modulus of elasticity of some commonly used materials. A scale of Mach number is included at a rate equivalent to maximum temperatures which may be encountered. Since the modulus of elasticity is proportional to the buckling load of an element, the curve shows the deterioration of load carrying capacity of the structure with temperature.

Another effect of high temperatures occurs as a result of the thermal gradients which occur within the component. Since various areas of the structure tend to expand at a rate according to the local temperature, a complicated stress pattern develops relative to the thermal gradients which exist. This would cause little problem if the temperature levels and gradients were such that materials remained truly elastic, but this is not the case. Behavior is inelastic in nature in some situations and creep deformations may occur in others.

Buckling of thin walled structural elements is probably the greatest danger in aircraft and missiles. This is due to the fact that elastic buckling of a wing panel can change the flow of air over the surface of a wing enough to produce a dangerous aerodynamic situation at high Mach numbers. The changes in flow pattern can then change the
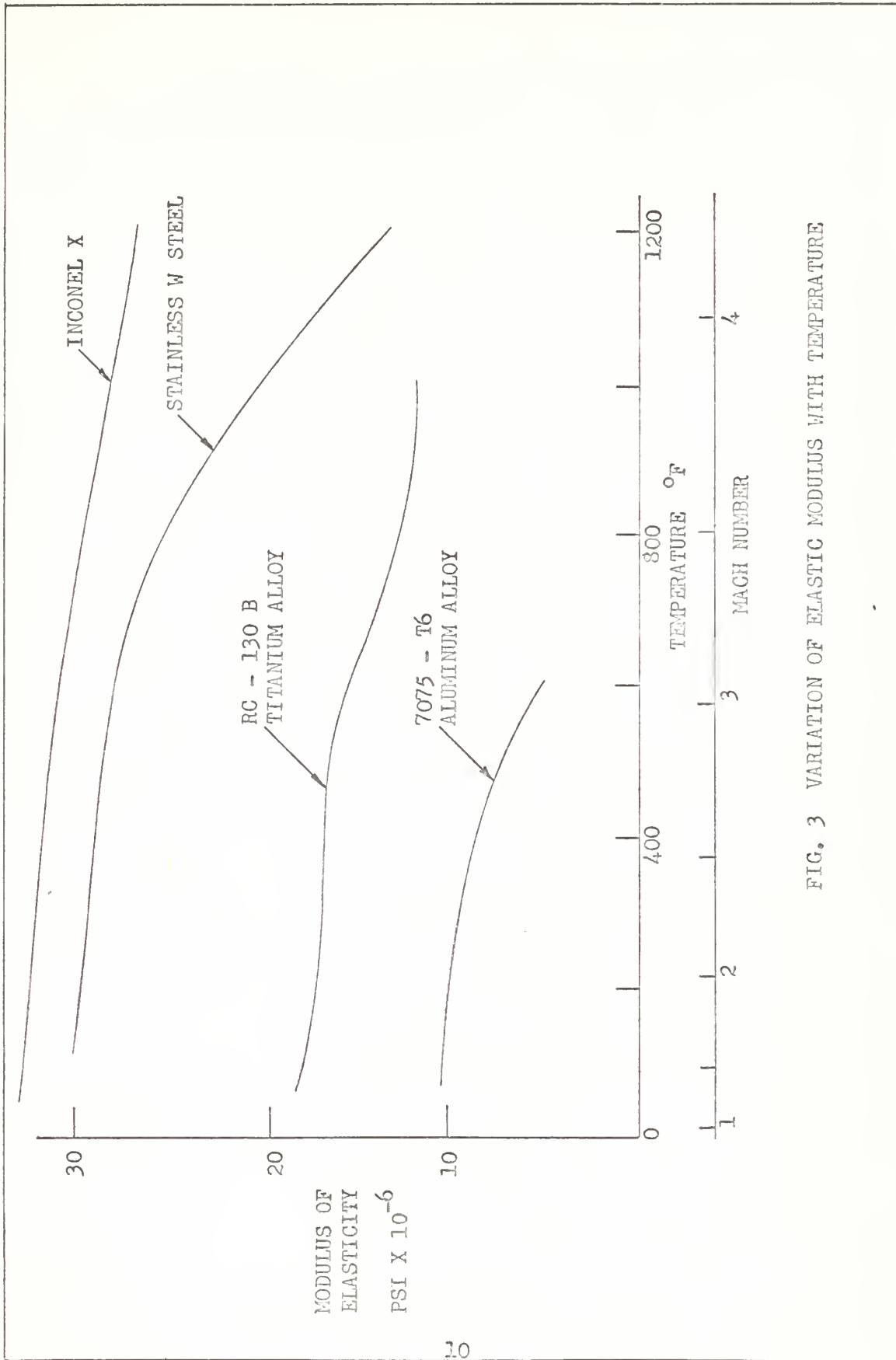
FIG. 3 VARIATION OF ELASTIC MODULUS WITH TEMPERATURE

INCONEL X

STAINLESS W STEEL

RC - 130 B
TITANIUM ALLOY

7075 - T6
ALUMINUM ALLOY

MODULUS OF
ELASTICITY
PSI X 10⁻⁶

TEMPERATURE °F

MACH NUMBER

30

20

10

0   400   800   1200

thermal gradient enough to produce new thermal stresses sufficient to cause inelastic buckling or failure of the element.

The mechanism of creep has been the object of considerable study in recent years. Creep may be defined as the time-dependent portion of the strain imposed upon a metal by the application of a stress. (3) A significant aspect of creep is that it is also temperature dependent. If a loaded component is above a critical temperature, it will undergo creep. This critical temperature is usually from 25 to 50 percent of the melting point of the material. For this reason creep can become a significant limiting factor in the choice of construction materials of vehicles which will undergo conditions of high temperature.

There are other effects of flight at high temperature, however, the most significant ones have been discussed here. The object of this section has been to emphasize the need for detailed study in this field. In the following sections it will be shown how needed experimentation in this field can be accomplished.

III. MODEL SIMILARITY

Due to the inherent difficulties associated with testing full scale structural components, it is often convenient to test a scale model. The problems associated with simulation of the general aerodynamic heating problem are the subject of several reports. (4, 5, 6, and 7) The authors of these reports vary somewhat in their approach to the problem, but they are unanimous in their conclusion that exact simulation of general aerodynamic heating by use of a scale model in an unheated wind tunnel is mathematically impossible. The alternative, therefore, to full scale testing is "incomplete aerothermoelastic testing". This is a testing technique in which some means other than an air stream is used to simulate aerodynamic pressure and heating. An example of this would be a facility in which the heat source were a radiant heater and the aerodynamic pressure source were a mechanical loading device.

Full development of similarity parameters as applied to thermal testing and mathematical proof of the impossibility of "complete aerothermoelastic testing" with a scale model may be found in Appendix A. "Complete aero-

thermoelastic testing" as meant here is chracterized by a test in which a scale model is tested in a wind tunnel with nothing but unheated tunnel air as a heat and pressure source.

The approach to the development of similarity parameters varies with the author. Dugundji (7) starts with the general equations of motion for a compressible, viscous, heat-conducting perfect gas. These equations are nondimensionalized and an order of magnitude analysis is made. The result is a series of combinations of quantities which must be the same for both the model and the prototype. These are called similarity parameters. Examples of these parameters include the well known Reynolds number, Prandtl number, and Mach number. Another method of obtaining the same results is by applying the Buckingham π theorem of dimensional analysis. This approach was used by O'Sullivan who wrote of his results for NACA in 1957 (4).

For reasons which will be elaborated on in following sections, the type of testing facility which will be proposed in this paper is one in which the source of heat is radiant and the source of loading is mechanical. Therefore

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a discussion of some similarity considerations for this type of testing will follow.

First it must be assumed that the adiabadic wall temperature and heat transfer coefficient or alternately the rate of heat input in an aerodynamic heating problem are known. These may be supplied by aerodynamic analysis or full scale tests. In order to simulate structural response (deflections, distortions, and thermal stresses) using a scale model, it is necessary to duplicate the variation of temperature in all directions in the model. In order to accomplish this, the quantity of heat input and heating rate must be scaled in a manner which can be described by similarity parameter laws. From a consideration of dimensional analysis, the applicable relationship can be expressed as: (8)

$$\frac{\delta}{L}, \quad \frac{\sigma}{E}, \quad \Delta T \alpha = f\left(\mu, \quad a, \quad \frac{E\alpha}{c}, \quad \frac{q\alpha L}{k}, \quad \frac{kt}{cL^2}\right)$$

where δ = deflection

L = chracteristic length

σ = stress

E = modulus of elasticity

ΔT = temperature increase

α = coefficient of thermal expansion

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μ = Poisson's ratio

a = absorptivity factor

c = volumetric specific heat

q = thermal flux

k = conductivity

t = time.

If it can be assumed that the same material is used for the model and prototype, the following relationships will automatically be true:

$$E_m = E_p \qquad\qquad \mu_m = \mu_p \qquad\qquad c_m = c_p$$

$$k_m = k_p \qquad\qquad \alpha_m = \alpha_p$$

where subscripts m and p denote model and prototype respectively. Furthermore, if $a_m = a_p$, then

$$\delta_m = \frac{L_m}{L_p} \, \delta_p$$

$$\sigma_m = \sigma_p$$

and $\quad T_m = T_p$

if the following conditions are satisfied:

$$q_m = \frac{L_p}{L_m} \, q_p$$

$$t_m = \left(\frac{L_m}{L_p}\right)^2 t_p \,.$$

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Therefore, for a ¼ scale model, $q_m = 4q_p$ and $t_m = t_p/16$.
The physical meaning of the first equality is that the
absorbed intensity per unit area in the model must be
four times a given intensity in the prototype to obtain
the same results. The second equation shows that, since
heating rate is proportional to t^{-1}, the heating rate of
the model must be 16 times larger than a heating rate in
a prototype to be simulated to have similar effects.

The above analysis shows the primary limitations of
testing small scale models. Maximum thermal stresses,
which are of primary interest, occur when temperature
changes are most rapid. It may be concluded, therefore,
that testing requiring large rates of temperature change
or high heating rates must be done on full scale or nearly
full scale models.

IV. THERMAL EFFECTS FACILITIES

The most accurate method for determining the behavior of an aircraft structure under the influence of aerodynamic heating would be a full scale flight test. For obvious reasons this is not practical. Therefore, the structure or component under consideration must be tested under conditions which most nearly simulate those which might actually be encountered. The difficulties and inaccuracies inherent in laboratory simulation of anything other than a full scale model have previously been discussed. There is an advantage to the laboratory test, however. This is the fact that data collection can be more complete in the laboratory, since a component of interest can be studied separate from the flight vehicle if necessary. The laboratory simulation of thermal effects can, therefore, be extremely useful for the evaluation of these effects. In this and following sections a survey will be made of equipment useful for the study of thermal effects in the laboratory.

Wind Tunnels

One of the most obvious approaches to the laboratory simulation problem might be the use of a wind tunnel.

There are several approaches to the problem of satisfying

the laws of similarity. These include the use of gasses

other than air or the use of materials other than those

of the actual component. In addition, since it is nearly

impossible to provide for flow of a magnitude that would

produce stagnation conditions representative of high

speed flight through a workable test section, some pro-

vision would be necessary for heating the circulating

gas. A design for such a wind tunnel is proposed by

Molyneux, (5) however he concludes that it is quite im-

practical.

Trussell and Weidman have described a recent endea-

vor by NASA to solve the problems described above on a

practical basis. Their approach was to equip the exist-

ing supersonic unitary plan wind tunnel at Langley

Research Center with an internal radiant heating device.

The test section of this tunnel is four feet high, four

feet wide and approximately seven feet long. The heat

source used was 96 quartz tube heat lamps mounted inside

the test section which provide about 26 BTU/ft^2 sec at a

distance of 12 inches from the heater. Surface tempera-

tures of up to 700°F are available. (9) Such a facility

is considered impractical for USNPS in view of the
fact that such a wind tunnel does not exist at USNPS
and projected needs do not warrant its construction.

An alternative which may be suggested is to equip
one of the present subsonic wind tunnels at USNPS with
some kind of heating apparatus. This could be accom-
plished by lining the heated area with metal to protect
the wood construction, and mounting a bank of quartz
tube lamps on each side. Such a setup would be suitable
for only specialized tests. The major restriction to
its use is the fact that the flow is subsonic. In add-
tion, the lamp banks must be mounted behind a heat
resistant glass enclosure to prevent interference with
the air stream. Resistance heating by passing a current
directly through the specimen may be a simpler method of
accomplishing the same results. In either case, care in
interpretation of results is necessary, since similarity
parameters must be satisfied.

Horton and Johnson have described a hypersonic wind
tunnel for structural research which exists at Stanford
University. This wind tunnel is of the standard blow-
down type with a test section variable in size of from
five to eight inches square and 14 inches long. The

largest model successfully evaluated was two inches in diameter. The heat source is a gas-fired heat exchanger through which air passes prior to passage through a hypersonic nozzle. (10) Although a blowdown tunnel exists at USNPS, conversion of this facility to a structural thermal effects study device would not be advised at this time in view of the cost involved and the limited use that it would receive at this institution.

Structural Testing Facilities

The wind tunnel approach to investigation of thermal effects on structures has been shown impractical. In addition, similarity considerations dictate the use of as large a scale model as possible for accurate correlation of results. Therefore, the only acceptable alternative is a full scale testing facility which would simulate aerodynamic heating with or without the addition of simulated aerodynamic forces. Such a facility has been built at Massachusetts Institute of Technology (8) and Stanford University (11, 12, 13). Testing facilities such as exist at these institutions can be as elaborate or as simple as needs and available funds prescribe. Investigations which can be conducted include:

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1. temperature gradient studies,

2. thermal stress studies,

3. combined static and thermal load studies,

4. combined dynamic and thermal load studies,

5. studies on variations in structural stiffness

   and strength at elevated temperatures,

6. cumulative thermal damage studies,

7. thermal fatigue studies,

8. contact joint conductance studies,

9. internal reradiation effects, and

10. deflection and distortion studies. (8)

The following discussion, therefore, will deal with this
type of facility in great detail.

Design Criteria

The design criteria of a facility for testing thermal
effects on aircraft structures are as follows:

1. The heat source must be such that high intensity
   rapid heating rates are available.

2. The heat input must be controllable in a manner
   that will permit simulation of a particular
   flight history of interest.

3. The facility must be large enough to permit test-

ing   ing of full scale or large scale model struc-

tural components.

4. Test specimen loading must be possible without

undue complexity.

5. The heating system should be such that it, by

itself, will not induce stresses into the

specimen.

6. The specimen should, if possible, be visible

during the test.

7. The entire system must be simple enough to be

operated by one or two persons.

8. Operation of the facility must be nonhazardous.

(8, 1)

In addition to these criteria, consideration must be

given to power, space, and money available for such a

facility.

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V. HEAT SOURCES

Radiant Heat Sources

Within the past fifteen years Abramson, of Research
Incorporated, Minneapolis, working under contract to the
Wright Air Development Center (14) and Horton at the
Royal Aircraft Establishment, Farnborough, England and
Stanford University (1, 15) have carried out considerable
research and development on systems of structural testing
at high temperatures. Their work had in common the use
of infrared radiation from quartz tube lamps as the heat
source. Abramson's work led to the marketing by Research
Inc. of the best commercially available "off the shelf"
components for this type of facility. Horton's work led
to the design and construction of a highly successful
facility of this type by three former USNPS students,
Campbell, Watson, and Geronime, at Stanford University in
1962 (11, 12, 13).

The T-3 quartz tube lamp, manufactured by General
Electric, is gas filled and consists of a tungsten fila-
ment held in the center of a quartz tube of 3/8 inch
outside diameter by a series of tantalum disc spacers.
Molybdenum foils are sealed into the ends of the quartz

tube and are attached to the filament wires. Power
output is of the order of 1 KW per lamp at 220 volts.
Up to 480 volts per lamp is permissible and under these
conditions 3 KW per lamp output is obtainable. Lamp
life is 5000 hours (16). Lamps may be mounted as close
together as ½ inch between centers in arrays. In such
a configuration power dissipation of 200 KW/ft^2 is
obtainable. (1) At the resulting heat flux density of
100 BTU/ft^{2}sec at two inches, specimen temperatures of
up to 2700° are available. (17)

Other approaches to heating by radiation have been
used with varying degrees of success. One approach is
the nickel-chromium heating element. A typical config-
uration is a 30 inch long element mounted at the focal
point of a parabolic reflector. Major disadvantages of
this system include high thermal inertia and low heat-
transfer rates. (18)

One additional method of radiation heating worthy
of note is the silicon-carbide Globar. These are resis-
tance type heaters mounted in a manner similar to quartz
tube lamps. This is the type of heating system origin-
ally installed in the facility at M. I. T. The primary
disadvantage of this system compared with the quartz

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tube again is high thermal inertia (resistance to temp-
erature change). In addition, tests conducted on the
M. I. T. facility showed a heat flux density of approxi-
mately 50 BTU/ft^{2}sec under conditions similar to those
in which a quartz tube lamp will produce 100 BTU/ft^{2}sec.
A further disadvantage is the fact that Globar elements
have a lifetime of up to 100 hours compared to 5000 hours
for the quartz tube lamp. (8)

Induction Heating Methods

A somewhat different approach to heating is the induc-
tion heating method. Briefly, this is accomplished by
inducing eddy currents in a test specimen made of elec-
trically conducting material by a changing magnetic field.
The resulting current flow in the material which has
resistance requires expenditure of energy, which appears
in the form of heat. In order to set up the necessary
magnetic field, the test specimen must be encircled by
an induction coil or alternately large pancake coils may
be mounted close to a surface to be heated. Advantages
of induction heating include high heating power density
(of the order 200 KW/ft^2) and low thermal inertia.

Considerable work was done by McLean and Lederman

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at Polytechnic Institute of Brooklyn in 1956 (19, 20)
on two small (1 KW and 20 KW) induction heating machines.
Models of wing structures approximately 8 inches by 12
inches in size were tested and results compared with
theory. Considerable satisfaction was expressed at results.
Types of tests which may be performed with this method are
somewhat limited since thermal gradients are only obtain-
able within the model itself. Heat application on the
outside of the model is constant due to the nature of the
required test setup. Experimental work with induction
heating on full scale models has proven the method effec-
tive but costly compared with radiant heating methods for
large scale applications.

Conduction Heating

Another method of heating which has been tested is
conduction heating. This is accomplished by covering the
test specimen with a heating blanket, strip heater or
film type heater. The heating blanket consists of elec-
tric resistance wires embedded in a flexible, electrically
nonconducting matrix such as rubber or plastic. A strip
heater is similar except that the heating element and
electrical insulation do not form an integral unit. A

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film type heater is formed by spraying the surface to be heated with an insulating material and then spraying on a series of electrical heating elements. Operating temperatures with these methods range from 450°F to 600°F. Power densities obtainable range up to 95 BTU/ft^2 sec (1 KW/ft^2) with 90% heating efficiency. (3)

Disadvantages of these methods as compared to others are many. First is the lack of high temperature capability (700°F as opposed to 2700°F for radiant heating methods). In addition, there is a certain lack of flexibility. All of these heaters must be tailored to the test specimen. In the case of film type heaters a semi-permanent attachment is necessary. A disadvantage with heating blankets is that sometimes hot spots result from the blanket not making intimate contact in an area. (18)

Resistance Heating

A heat source useful for some applications is the heat of a heavy electrical current passing through the test specimen itself. This technique is particularly appropriate to the testing of materials in a universal testing machine for reactions to combined applied loads and thermal stresses. An advantage to be gained with this type of heating is the fact that the test specimen

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is fully visible throughout the entire test. A restriction is the fact that the test specimen must be an electrical conductor.

Conclusions

Three conclusions can be drawn from this discussion of heat sources. They are:

1. For a full scale structural testing facility the most effective heat source is the quartz tube radiant heating lamp.

2. For a small testing unit with limited application an induction heating device yields satisfactory results.

3. Resistance heating is very effective for universal testing machine applications when possible.

Table I shows a comparison of the various types of heating sources.

TABLE I COMPARISON OF HEAT SOURCES

	MAX TEMP OF SOURCE °F	HEAT UP TIME °F PER SEC	HEATING RATES BTU/FT^2-SEC	ADVANTAGES	DISADVANTAGES
RADIENT SOURCES					
QUARTZ TUBE LAMPS	5800	10,000	100	1. OPERATES AT ANY TEMP 2. LOW THERMAL INERTIA	1. EXPENSIVE
HEATING ELEMENT	2000-2400	56	15	1. INEXPENSIVE	1. HIGH THERMAL INERTIA
GLOBAR	3000	100	50	1. LONG TIME OPERATION AT HIGH TEMP POSSIBLE	1. HIGH THERMAL INERTIA
INDUCTION	1000	1000	100	1. LOW THERMAL INERTIA	1. EXPENSIVE 2. ELECTRICAL CONDUCTOR SPECIMEN 3. THERMAL GRADIENTS IMPOSSIBLE
CONDUCTION					
HEATING BLANKETS	450	50	1	1. LOW POWER REQUIRED	1. LOW TEMP CAPABILITY 2. HIGH THERMAL INERTIA 3. LACK OF FLEXIBILITY 4. HOT SPOTS
STRIP HEATERS	600	50	75		
FILM TYPE HEATERS	500	50	25		
RESISTANCE HEATING	MELTING	50-100	VARIES	1. SPECIMEN FULLY VISIBLE	1. ELECTRICAL CONDUCTOR SPECIMEN 2. NO THERMAL GRAD

VI. POWER REGULATION

One of the basic needs of any heating facility is a method of regulating electric power input to the heating devices such that desired temperatures and heating rates are achieved. With a full scale facility, a 750 KVA, 480 volt, 3 phase transformer such as is available at USNPS is capable of producing over one megawatt of power. Heavy duty equipment is necessary to regulate this power. Methods available include the variable transformer, saturable core reactors and electronic devices. These methods will be discussed in this section.

Variable Transformers

Possibly the most direct approach to power regulation is a variable transformer. Advantages of this system include low cost and smoothness of regulation. The primary disadvantage is the relative slowness of operation. In addition, its bulkiness makes the variable transformer inferior to other methods of power regulation for a radiant heating facility. (15)

Saturable Core Reactors

A more effective method of power regulation and one which has been used extensively is the saturable core

reactor. This is essentially a d-c polarized choke inserted in the primary circuit of a transformer. When premagnetization is increased by passing current through the d-c winding, the a-c resistance of the choke is reduced and power output increases. This system is relatively trouble free and characterized by long life. Disadvantages include bulkiness and relatively slow response rate. It is this type of regulation which is incorporated in the system at Stanford University. (11)

Electronic Methods

A modern method superior in all respects to others is the use of electronic components such as ignitrons, thyratrons or silicon controlled rectifiers (SCR) in a combination regulation-control device. Advantages of these systems include extremely high response, compactness, accuracy, and reliability.

An ignitron is a large metal vacuum tube in which a pool of mercury is the cathode and a graphite cylinder the anode. A third electrode, the ignitor, is a short bar of refractory material in contact with the surface of the mercury. Current flows when the anode is positively charged and ionized mercury vapor exists in the space between the anode and cathode. The positive half of an

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applied a-c current supplies the first requirement.
The mercury vapor is supplied by energizing the ignitor
with a voltage of 100-200 volts. Current then starts
flowing and continues to flow until the end of the half
cycle. By varying the time at which the ignitor is
"fired" in each cycle, the amount of power per half cycle
is varied. Since current flow in a single ignitron takes
place only during a half cycle it is necessary to connect
two of them in inverse parallel for full wave output. A
chracteristic of this device is that 100% overload for
periods of up to five minutes is permissible. This and
other advantages make this device the best high response,
heavy current power regulating device on the market today. .
(17)

    For lower current loads a thyratron system is recom-
mended. This device operates in much the same manner as
an ignitron except that in a thyratron the electrons are
emitted by a heated dathode and current flow is "trigger-
ed" by a grid. Thyratrons do not have the short term
overload capacity that characterizes ignitrons. Advan-
tages include low cost, light weight and compactness.
For multichannel applications thyratrons are recommended

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provided loads greater than 40 amps per channel are not required. (17)

For applications requiring loads greater than 40 amps but not great enough to require the use of ignitrons, the silicon controlled rectifier is useful. This is a solid state device which works in much the same manner as the ignitron and thyratron. For applications in which loads are not too great it has an advantage over other methods in that no external cooling devices are necessary. In addition, it is extremely compact, lightweight and maintenance free. (17)

Conclusions

The following recommendations are made concerning the use of power regulating devices.

1. For heavy power output applications ignitron regulation units are recommended.

2. For medium power output applications SCR units are recommended.

3. For light power output applications thyratron regulation is recommended.

4. For high current low power applications in which high response is unnecessary variable transformer regulation is satisfactory.

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VII. SYSTEM CONTROL

System control must be such that each part of the test specimen is held at the desired temperature. In Section II it was shown that in an aerodynamic heating problem, the desired temperature is high at the leading and trailing edges of a lifting surface and lower in the midchord area. It was pointed out that the heat flow into the structure may be expressed as:

$$q = h(t_{aw} - t_s) - \lambda t_s^4 + f(S)$$

where

λt_s^4 = the Stefen-Boltzman radiation loss to free space ($BTU/hr/ft^2$)

f(S) = solar radiation gain $\sim$ 400 $BTU/hr/ft^2$.

There are two approaches to controlling the heat input into a test specimen which corresponds to these conditions. The first is to compute analytically a temperature profile in the specimen which varies according to a flight profile being simulated. This temperature profile is then fed into an electronic controller which which compares program temperature with specimen temperature as measured by thermocouples welded to the test specimen. Error signal is transmitted into the power

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regulator which varies power input into the heaters.
Fig. 4 shows a block diagram of this system. This
system could consist of as many channels as necessary
to simulate the desired temperature profile over a
surface such as a wing. The primary advantage of this
system is simplicity. In addition, since temperature
is measured directly at the specimen, the system is
independent of the source of heat. A disadvantage of
this system is the fact that the theoretical tempera-
ture to be programmed into the control system is ex-
tremely difficult to compute accurately. (1)

The other method is to program each of the quanti-
ties in the heat flow equation as a function of time
into a computer. The output of the computer is heat
flow rate corresponding to a flight profile to be simu-
lated. Net radiant heat flow rate at the test specimen
is measured with special transducers which compare
radiant heat to and from the test specimen. This system
has the advantage that no control system connections or
thermocouples are necessary on the test specimen itself.
This might be useful, for example, in the measurement of
the effects of solar radiation on an orbiting satellite.

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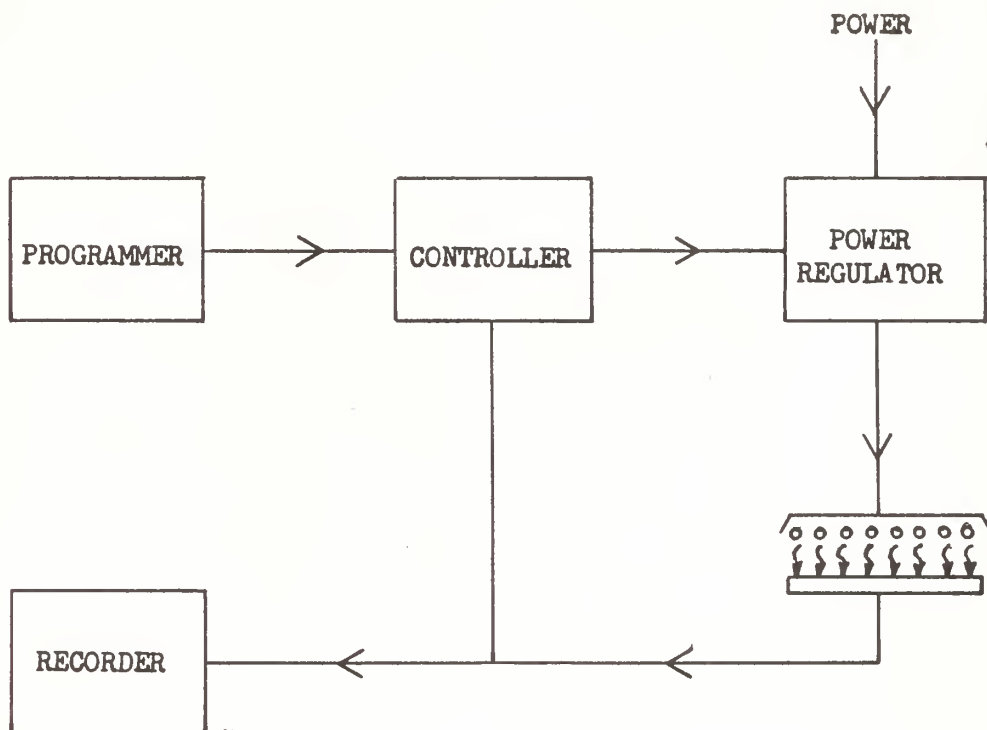
FIG. 4

BLOCK DIAGRAM OF TEMPERATURE CONTROLLED HEATING SYSTEM

The disadvantage of this system is the uncertainty of how much heat actually is transmitted to the test specimen. The transducer only measures net radiation to the test specimen; therefore, convective heat is unaccounted for.

The decision as to which system to use must be made primarily on the basis of application. Research Incorporated offers a variety of programmers for use by either system. They also offer a heat flux density sensing device which would replace thermocouple input to the system with no other component changes. Therefore, a system built of these components would be most flexible.

The control system on the facility at Stanford, was proposed by Professor W. H. Horton (1) and the prototype was designed and built by Lts. Watson, Campbell, and Geronime (11, 12, 13) It is based on a combination of the two systems. An additional refinement is included as a hybrid computer function generator with a time sharing feature so that each of four heating channels share its use. Feedback is by both temperature from thermocouples in the test specimen and transducers in the radiant heat path. Fig.5 shows a block diagram of this system. (11)
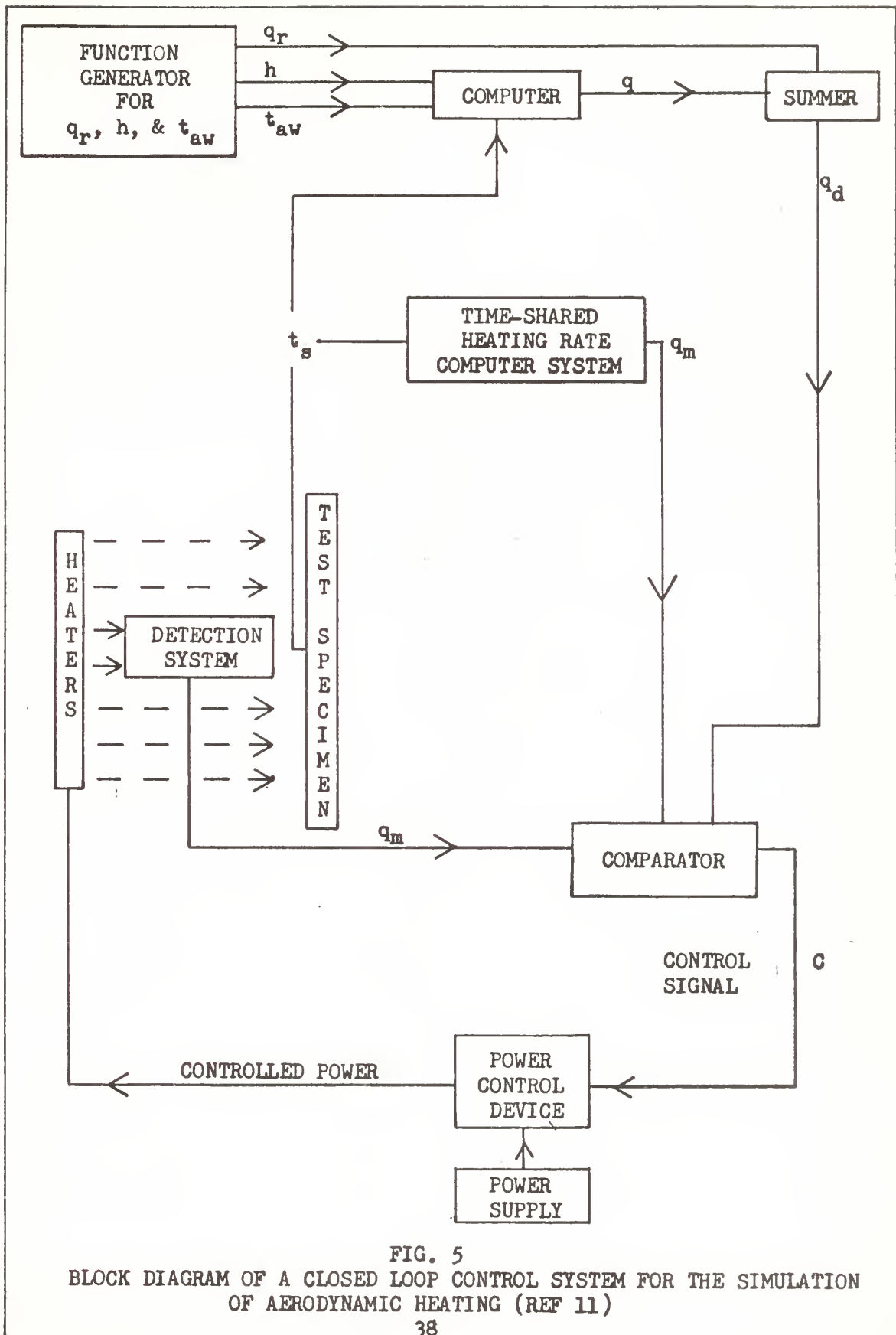
FIG. 5
BLOCK DIAGRAM OF A CLOSED LOOP CONTROL SYSTEM FOR THE SIMULATION
OF AERODYNAMIC HEATING (REF 11)

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The unique feature of this system is the digital
to analogue converter. The premise on which this sys-
tem is based is that q (the heat which is conducted at
the surface) is the actual quantity of interest since
it combines the effects of all inputs and losses. The
computer, therefore, reads digital values of T_{aw}, h,
and f(S) since these are the basic given time dependent
variables. It then computes q in a basic analogue com-
puter utilizing measured values of T_S. A significant
cost saving feature is the fact that a single amplifier
per channel is used for all operations. This is done by
storing each computed value as a voltage in a capacitor
until the next one can be computed and then recalling
the stored information for the summing process.

The advantage of this system is that the three vari-
ables, T_{aw}, h, and f(S) can be varied independently
according to the flight profile to be simulated. Also,
there is no need to precompute the T_S which would corres-
pond to a given flight condition. Although there are no
disadvantages to this system, it is not available com-
mercially. The advantages to be gained are not considered
to be sufficient to warrant its construction. The control

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system which is recommended for USNPS, therefore, is
the more versatile temperature controlled system which
is commercially available.

VIII. ASSOCIATED EQUIPMENT

There are numerous types of equipment which must be used in conjunction with any thermal effects testing setup in addition to the major components already discussed. In this section some of these miscellaneous equipments will be discussed.

Reflector Assemblies

The first requirement of a reflector assembly is that it reflect a maximum amount of the radiant heat energy in the direction of the test specimen. Fig. 6 shows a comparison in this respect of several materials which have been used for this purpose. (1) Note that the most efficient metal is gold followed by aluminum in various forms.

A second requirement of the reflector material is that it be capable of withstanding the temperatures to which it will be subjected. Again gold is by far the best material. Research Inc. offers reflectors plated with a special "Ceragold" (Fig. 7) which will withstand any temperature within the capability of the quartz tube lamps themselves. It is this reflector type which should be used for the most intense heating applications.

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FIG. 6 VARIATION OF MATERIAL REFLECTIVITY (REF 1)

1. Electrolytic Gold
2. Vapourised Aluminium
3. Polished Aluminium
4. Alzac Aluminiun
5. Chromium Plating
6. Relative Energy Distribution
   Of A Typical Infra Red Lamp

Reflectivity %

Visible

Infra Red

Wave Length Angstrom Units

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FIG.   _____ INC. REFLECTOR ASSEMBLY
        WITH LAMPS
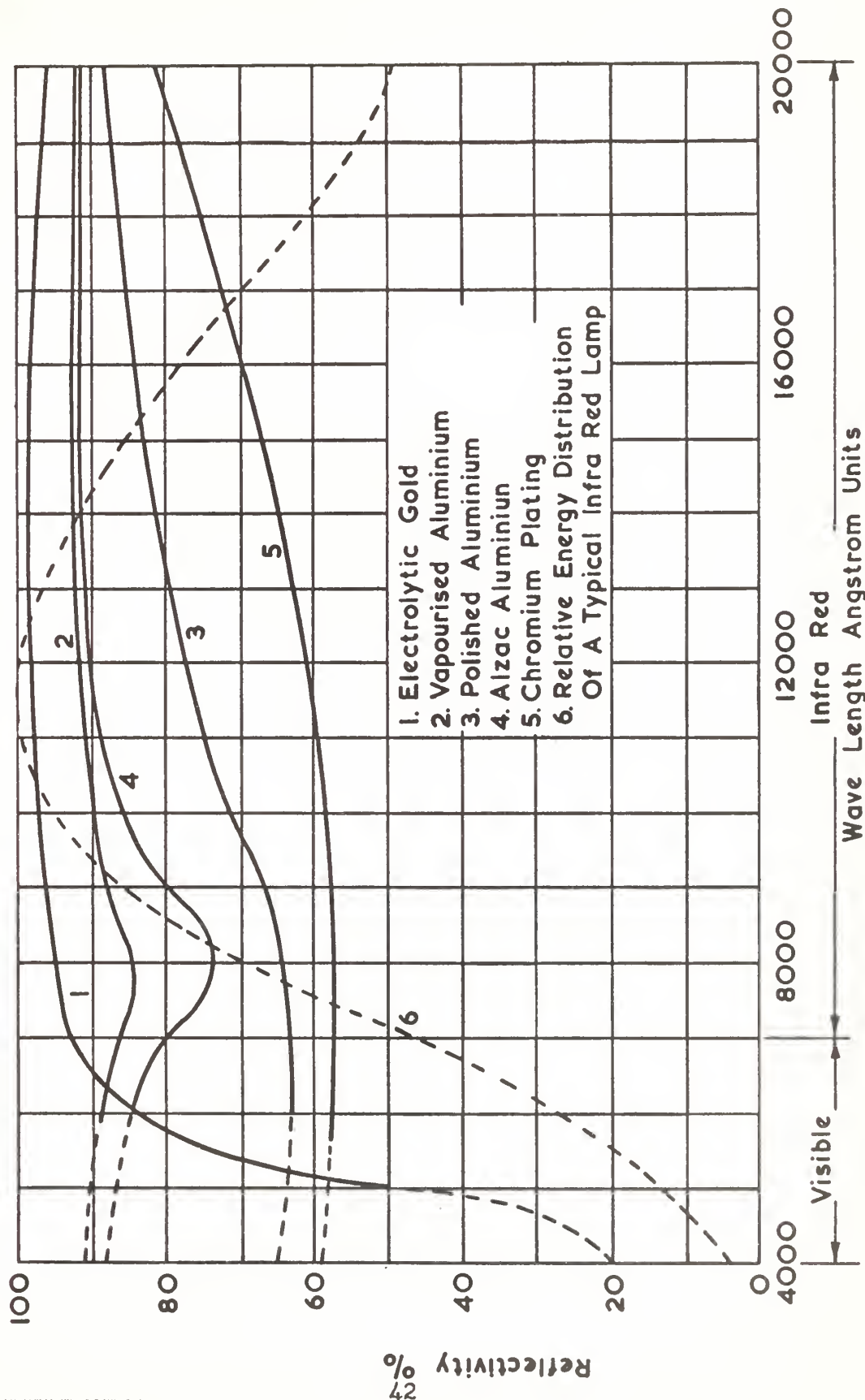
Some concern was expressed recently during a test by Lt Schultz while working on a thesis project at USNPS, when it was discovered that upon occasion, some gold was deposited on the test specimen from the reflector. (21)   This reflector was Research Inc. model AU8-618B.   It was concluded by Lt Schultz after analysis that this occured only when heating titanium with the reflector accidentally in contact with the specimen. It was also concluded that since this occured during use of a new reflector, the source of the gold might have been some residue on the new reflector.   Care should be taken in this respect, therefore, when using gold reflectors.

An alternate reflector material offered by Research Inc. is polished aluminum.   Already mentioned was the fact that aluminum is less efficient than gold in reflective ability.   In addition to this drawback, aluminum reflectors will not withstand nearly as much heat as the gold reflectors. For example, if maximum power were inadvertantly applied to lamps in an uncooled aluminum reflector, it would take about one minute to melt! Since replacement cost is about $50 per reflector, uncooled

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polished aluminum reflectors should not be used where the ambient temperature exceeds 700°F. (17)

Since gold reflector assemblies are about 50% more expensive than aluminum reflector assemblies, a third alternative is offered by Research Inc. in the form of water-cooled aluminum reflector assemblies. This enables heat output to be considerably increased over unheated aluminum, but not to the point that full power on the lamps would not melt the reflectors. In addition, a heat exchanger and water pump with connecting tubing must be provided. For a small facility, the additional expenditure for this equipment would increase the cost of this cooled aluminum reflector unit to a point beyond that of gold reflectors.

In view of the preceeding discussion the following conclusions can be drawn:

1. For high intensity heating requirements, Ceregold plated reflectors are recommended.

2. For large facilities involved with high radiant flux densities (up to 200 KW/ft^2 with specimen temperature up to 2700°), water-cooled aluminum reflector assemblies are recommended.

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3. For smaller facilities which require lamp
    temperatures over 700°F, Ceregold plated
    reflector assemblies are recommended.

Quartz Tube Supports

The weakest point in the quartz tube lamp is the
end seal. In fact, for high temperature applications,
it has been shown that lamp life is directly propor-
tional to end seal temperature. (17) For this reason
manufacturers of reflector assemblies offer an end
seal aircooling system as an option. Inclusion of
this device is recommended for any but low temperature
application thermal testing facilities.

Efficiency Aids and Temperature Measurement

In order to provide maximum efficiency of power
radiation, the specimen should be painted black. This
is due to the basic law of physics that a black body
absorbs heat better than any other. No recommendations
were available by commercial manufacturers of such heat
resistant black paint at this writing. Watson, however,
reports that he tested several commercial products for
such use and found the best to be common stove blacking.
(13) This paint was reported extremely resistant to
heat, easily applied by brush, and resulting in an even

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coat. A disadvantage is that the coating is easily
scratched.

Standard thermocouples are available for tempera-
ture measurement. Chromel-Alumel thermocouples are
available for use at temperatures up to 2400°F. Good
response was reported by Watson using this type at
temperatures up to 2450°F (13). For measurement of
temperatures beyond this limit, use of the more expen-
sive platinum-rhodium thermocouple is required.

High Temperature Strain Gages

For many applications of a thermal testing facility
it will be required to measure strain. Several prob-
lems are present in this regard. Some of these prob-
lems and limitations will be discussed here.

For temperatures up to 350°F phenol-resin (Bakelite)-
backed gages with either Bakelite or high-temperature
epoxy cement bonding are used. Few insurmountable prob-
lems are encountered in this range. Bonding and temp-
erature compensation procedures are well established.

In the temperature range of 350°-600°F mounting
techniques are different in that a different gage carrier
material is used. Since Bakelite becomes unstable at
350° F, phenol-resin or glass fiber bases are used.

These are good up to 450°F when bonded with a suitable cement (EPY-400, Mithra 200, etc). For applications between 450°F and 600°F the copper-nickel filament must be imbedded directly in high-temperature organic cement (filled epoxies, phenols, and epoxy-phenol combinations). Gage readings must be corrected for gage resistance change and gage factor change. (22)

Above 600°F, accurate measurement of static strain is very difficult. Since copper-nickel alloys become unstable at 600°F, other alloys are used for the filament. Chrome-nickel alloys (Nichrome V as marketed by Driver-Harris Co.) are often used because resistance and gage factors tend to remain more stable than do these properties with any other alloy at temperatures up to 2000°F. However, since these properties do change with temperature, many corrections must be applied and consistant accurate results are difficult to attain. (22)

In addition to filament problems there are serious bonding problems above 600°F. The most common method is to imbed the gage element in a ceramic cement (metallic oxides with a phosphoric acid binder for chemical setting or refractory materials which are flowed in place).

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Commercially available ceramic cements for this purpose are: Budd-Type H; Allen P-1 and PBX-Robert A. Allen Co., Mechanicsville, N. Y.; Brimor U-529- Morganite, Inc., Long Island City 1, N. Y. These cements have been used in temperatures up to 1500°F, although there is a strong tendency in upper ranges toward decreasing resistance with increasing temperature. (22)

One other method of gage mounting for use at high temperature worthy of note is a process developed by Rolls-Royce. This process utilizes oxide rods which are melted by oxyacetylene flame. The atomized molten oxide is sprayed on the pre-positioned gage to bond it in place. Good bonding and thermal-stability characteristics up to 2200°F were obtained through the use of laminar base coatings. (22) Equipment for this process is available through BLH Electronics (a division of Baldwin-Lima-Hamilton). The spray gun costs about $1000 and supporting equipment such as gas tanks, flow meters, etc. costs about $700. Therefore, due to the high cost and limited applications, purchase of this equipment is not recommended at this time.

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IX. RECOMMENDATIONS FOR USNPS

In the previous sections the experimental study
of thermal effects on aircraft structures has been
discussed from all aspects in general terms. In this
section these concepts and conclusions will be used
in an effort to design a facility for the investiga-
tion to thermal effects on structures by students at
USNPS. Several alternate facilities will be proposed
and cost figures will be included.

Facility Placement

Fig. 8 shows a plan view of the basement of Build-
ing 234 where the proposed facility is to be located.
Three power sources are shown as load centers T-6,
T-7, and T-9. Load center T-6 supplies heat and lights
for the building. Load center T-9 services the mechani-
cal engineering laboratory including a linear accelerator.
Load center T-7 supplies power for the wind tunnel and
structures laboratory. The load floor, where heavy
mechanical forces may be applied to a test specimen, is
shown.

In the placement of a radiant heating facility, the
primary consideration must be the source of electrical
power. Load center T-7 must be the source of power for
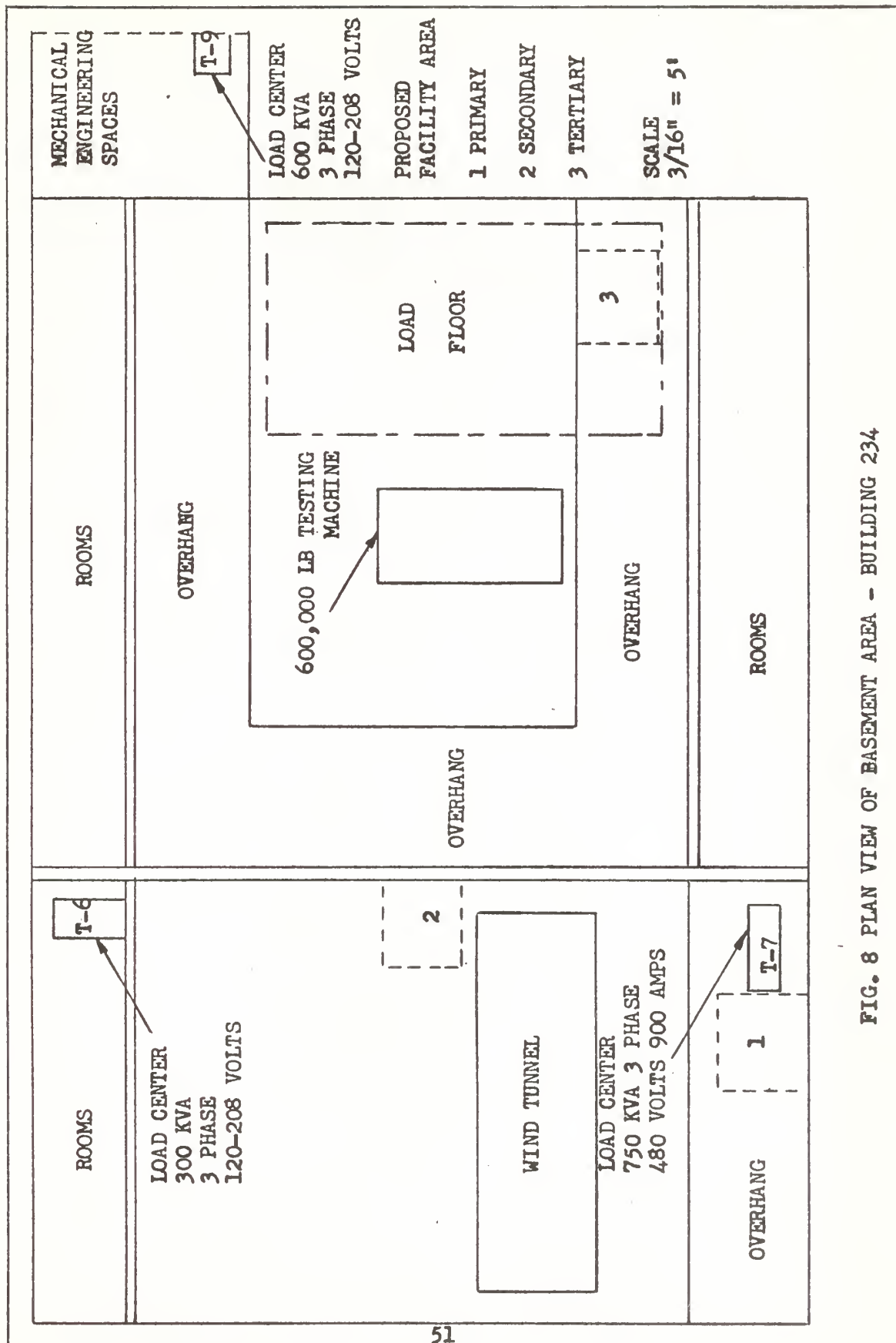
FIG. 8 PLAN VIEW OF BASEMENT AREA – BUILDING 234

MECHANICAL
ENGINEERING
SPACES

T-9

LOAD CENTER
600 KVA
3 PHASE
120-208 VOLTS

PROPOSED
FACILITY AREA

1 PRIMARY

2 SECONDARY

3 TERTIARY

SCALE
3/16" = 5'

ROOMS

OVERHANG

LOAD
FLOOR

3

600,000 LB TESTING
MACHINE

OVERHANG

OVERHANG

ROOMS

T-6

LOAD CENTER
300 KVA
3 PHASE
120-208 VOLTS

2

WIND TUNNEL

LOAD CENTER
750 KVA 3 PHASE
480 VOLTS 900 AMPS

1

T-7

ROOMS

OVERHANG

this system for two reasons. First, as shown on Fig. 8, it has 3 phase, 750 KVA, 480 volts, and 900 amps available, which is more than either of the others. Second, the other load centers are constantly in use, whereas load center T-7 is only used for testing by the Aeronautics Department. There will be applications when nearly the entire capacity of the load center will be useful. In these cases all other use of the load center must cease and this will best be arranged with the users of load center T-7.

Another consideration in the placement of this testing facility is the fact that certain applications will require loading the test specimen. This is most easily done on the load floor since both test rig and loading device can easily be bolted to the floor. If it is desired to load a specimen other than on the load floor, a simple loading platform can be constructed such that it mounts both test specimen and loading device as shown in Fig. 9. Such a device can be built by laboratory personnel for the cost of the material, which is estimated to be $1000.

The proposed testing facility will occupy about 225 square feet. The primary area proposed is immediately

FIG. 9

LOADED HEATED SPECIMEN IN SELFCONTAINED LOADING JIG

REFLECTOR ASSEMBLIES

TEST
SPECIMEN

MOUNTED AS
RIGID
CANTILEVER

REFLECTOR ASSEMBLIES

HYDRAULIC
JACK

LOADING PLATFORM
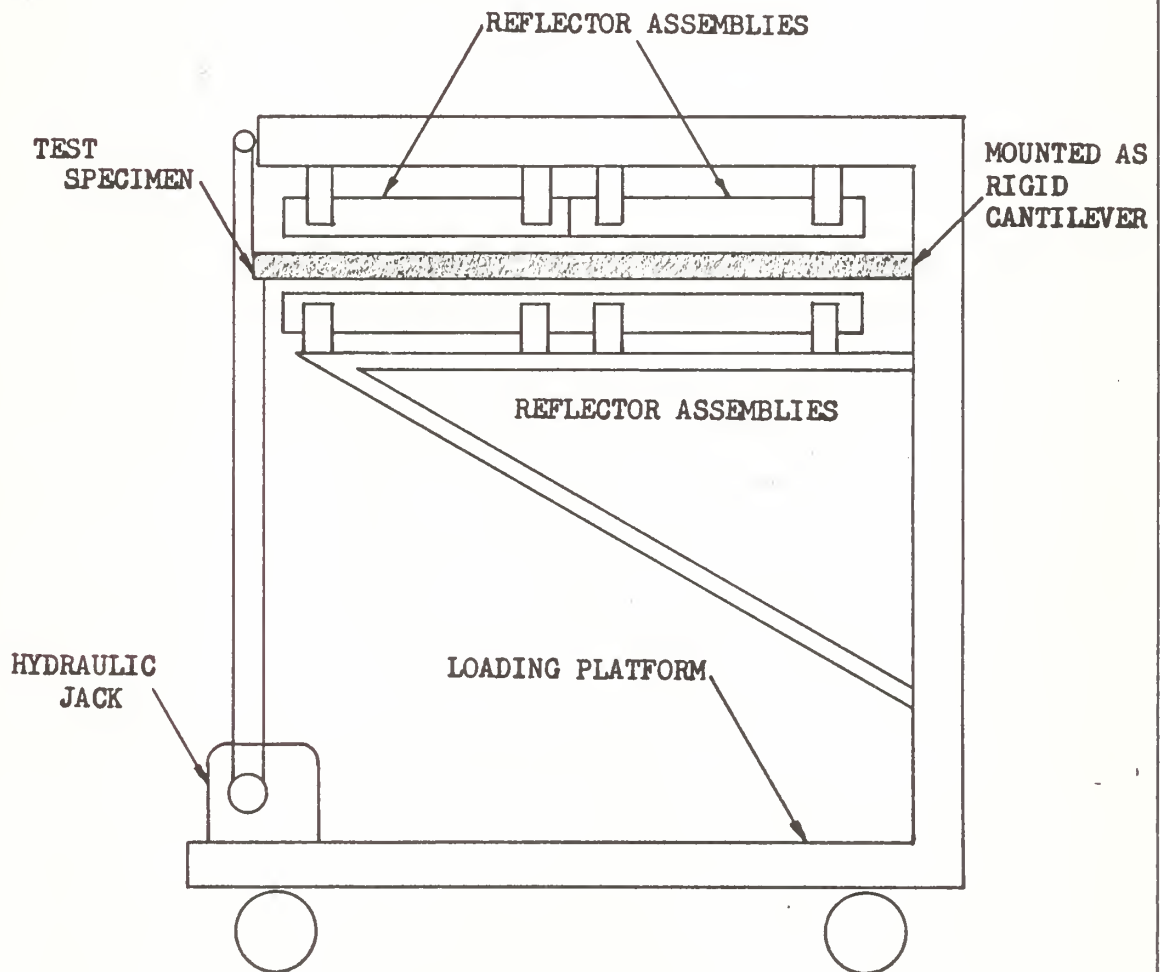
adjacent to load center T-7 as shown in Fig. 8.
The main reason for this choice is the fact that it
was estimated by Public Works engineers that it would
cost about $8000 to extend the entire power capability
a distance of 100 feet. It would only cost $1000 to
build the loading platform. The floor at this loca-
tion is concrete with built-in trenches which could
be used for power and control cables. An additional
advantage is that water is located here. This will be
necessary for ignitron power regulator cooling.

A secondary location is also shown on Fig. 8. This
is about 50 feet from the load center and the cost of
extending the required power to this area is estimated
at about $4000. An additional $1000 would be needed to
install a proper concrete floor since none exists here
now. Use of this area would, therefore, cost $5000
more than use of the primary area.

A tertiary location for the facility, as shown in
the figure is on the load floor of the structures lab-
oratory. This would eliminate the cost of a loading
platform; however, the power extension cost would be
about $8000 for a net increase of $7000 in overall cost.

Regardless of the location, it should be pointed out that when this facility is used at or near its capacity, no other major power requirement may be served by the same load center. Under these conditions, therefore, it will be impossible to run the wind tunnel and the thermal effects testing facility at the same time. This should pose no major problem, however, since run time for a typical test with this facility will not exceed ten minutes. An exception to this may be creep tests; however, these will no doubt be at relatively low temperatures (500°-800°F), thus requiring an insignificant amount of power.

Equipment Proposals

An investigation was made of the commercial availability of suitable "off the shelf" equipment of the type needed for a thermal effects testing facility and it was found that at least one company markets it. Prices quoted here will be approximate, therefore, and will be based on the current known prevailing prices on known components.

In order to show how much testing can be done with a given amount of money, a hypothetical set of general

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specifications for a proposed facility will be assumed.
These are as follows:

1. The three phase, 750 KVA, 480 volt, 900 amp
    load center (T-7) will be the only source of
    power.

2. The heating source will be infrared quartz
    tube lamps.

3. $2000^{\circ}F$ will be the maximum specimen tempera-
    ture required. Reference to Figs. 1 and 2
    show that this simulates speeds up to Mach 5
    at 50,000 ft.

4. Three channels will be sufficient to obtain
    temperature profiles desired in test specimens.

5. A test specimen with an area of nine square
    feet is to be tested. An example might be a
    missile fin model 1.5 ft by 3 ft to be heated
    equally on both sides.

6. A capability to program a time temperature
    variation into each channel is desired. Such
    a capability is necessary to simulate a flight
    in which a wide variation of conditions are
    met in a relatively short time.

In addition to these general specifications for the overall facility, each component should also have certain capabilities which will be listed.

Reference to Fig. 4 shows that the primary components necessary for each channel of such a system include: 1. a programmer, 2. a controller, 3. a power regulator, 4. lamp banks, 5. a feedback device, and 6. a data recorder. Some of these components can be combined into integral units. If available these are to be preferred over individual components since the combined unit is more compact and the components are matched to each other, thus reducing sources of error in temperature control.

One type of combination unit which would be both versatile and economical for the facility proposed is a combination temperature controller and power regulator for all three channels. It should meet the following specifications:

1. Three independently controlled and regulated channels.

2. Operate interchangeably on 230 or 460 volts.

3. Rated current capacity of 300 amps per channel.

4. Power output of 130 KW per channel at 460 volts.

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5. Overload capacity of twice rated current.

6. High response to feedback signal.

7. Precise temperature control ($\pm \frac{1}{4}$% of full scale).

8. Zero to full power capability.

9. Proportional band control (ability of operator to select highest system accuracy commensurate with system stability).

10. Corrective signal limiter control (overvoltage protection for lamps).

11. Selection of manual or remotely programmed control signal.

12. Portable, compact, lightweight.

A unit which meets the above specifications currently sells for about $8000.

Assuming that the power regulation and signal control functions for the three channels are taken care of, a second combination unit which can be added immediately or at a future date is a programmer. It is definitely advisable to have all channels programmed by a single unit to facilitate coordination of the three channels. A single unit will turn the program for all channels on and off simultaneously or singly as desired. The

following specifications must also be met:

1. Program preparation easily and quickly accomplished by an unskilled operator.

2. Static accuracy and repeatability of at least 0.1%.

3. Dynamic accuracy of 1% at maximum signal rate of change.

4. Adjustable time base from 10 seconds to 10 minutes.

A unit which meets the above specifications currently sells for about $3500.

For the actual heating process quartz tube reflector assemblies are necessary. For reasons previously stated, it is recommended that units with gold reflectors and air-cooled lamp-end seals be utilized. Size of reflector units is variable; however, for the example test specimen cited (missile fin 1½ ft by 3 ft), coverage could be accomplished by 12 units 6 inches by 18 inches in size. This would cover both sides completely. Since the Aeronautics Department already owns four such reflectors, purchase of eight additional units is necessary. Cost is about $80 per unit for a total of $640.

Quartz tube lamps come in all sizes and power ratings. As will be shown later, the proper lamp is General Electric lamp number 1600T3/1CL/HT, or its equivalent. This is rated at 1600 watts at 240 volts. Twice rated voltage is permissible, however, and in this condition power output is 4.5 watts. Cost per lamp is about $10. Since each reflector requires eight lamps, a total of 96 lamps is needed. The Aeronautics Department presently owns 32 lamps; therefore, only 64 are needed. Total cost then of the 64 lamps is $640.

Separate from the heating facility itself but still a vital part of any investigation is a method of data collection. A means must be available to read thermocouple data as well as strain gage data for any thermal stress problem. Furthermore, the readings must be continuous for any but a constant temperature test. For this purpose it is recommended that one or more multichannel strip chart recorders be purchased. Two eight channel recorders would pro-vide for independent temperature and corresponding strain measurement at eight positions in the hypo-

thetical missile fin previously mentioned as a possible test specimen. Each recorder will cost about $3000 for a total of $6000. Prices quoted are for constant reading type recorders complete with amplifiers.

Additional equipment necessary to complete the facility includes thermocouples, mounting frames, and necessary wiring. Cost of these items will vary greatly, but an estimate of their combined cost is $2000.

Now that the basic components have been described, let us see how the assumed specifications are met by this facility. The basic requirement was that a surface temperature of $2000^{\circ}F$ at the specimen depends on a number of things, including heat absorption ability of the specimen and heat losses at specimen. A typical value, however, for a black painted aluminum specimen is 32 KW/ft^2. Assuming a typical heating rate efficiency of 67%, it is found that 48 KW/ft^2 at the heat source is required. Since each reflector module is six by 18 inches in size (3/4 ft^2), and holds eight lamps, it is readily seen that each lamp is required to produce 4.5 KW. This is the maximum output per lamp at 480 volts for the lamps recommended. Since

nine square feet must be heated, the total power output required is 432 KW. The controller-regulator unit recommended is rated at 130 KW per channel for a total of 390 KW available. This can be doubled for periods of five minutes and under these conditions could easily handle the 432 KW required. If any part of the test specimen requires a lower temperature, or if heating efficiencies better than those predicted prevail, the specified temperature could be held for a longer period of time. The amperage requirement for 432 KW at 480 volts can be seen to be 900 amps, which is the maximum available from load center T-7. Therefore, the requirements as stated in the assumed specifications are just met by this proposed facility.

System Layout

Fig. 10 shows a proposed system layout. If location one as shown in Fig. 8 is chosen, the south wall would be the top boundary in Fig. 10. The load center would therefore border on position one in the figure. The controller-regulator unit should be as close to the power outlet as possible (position two) so that attachment cables may be short. The recorders and

FIG. 10

SYSTEM LAYOUT
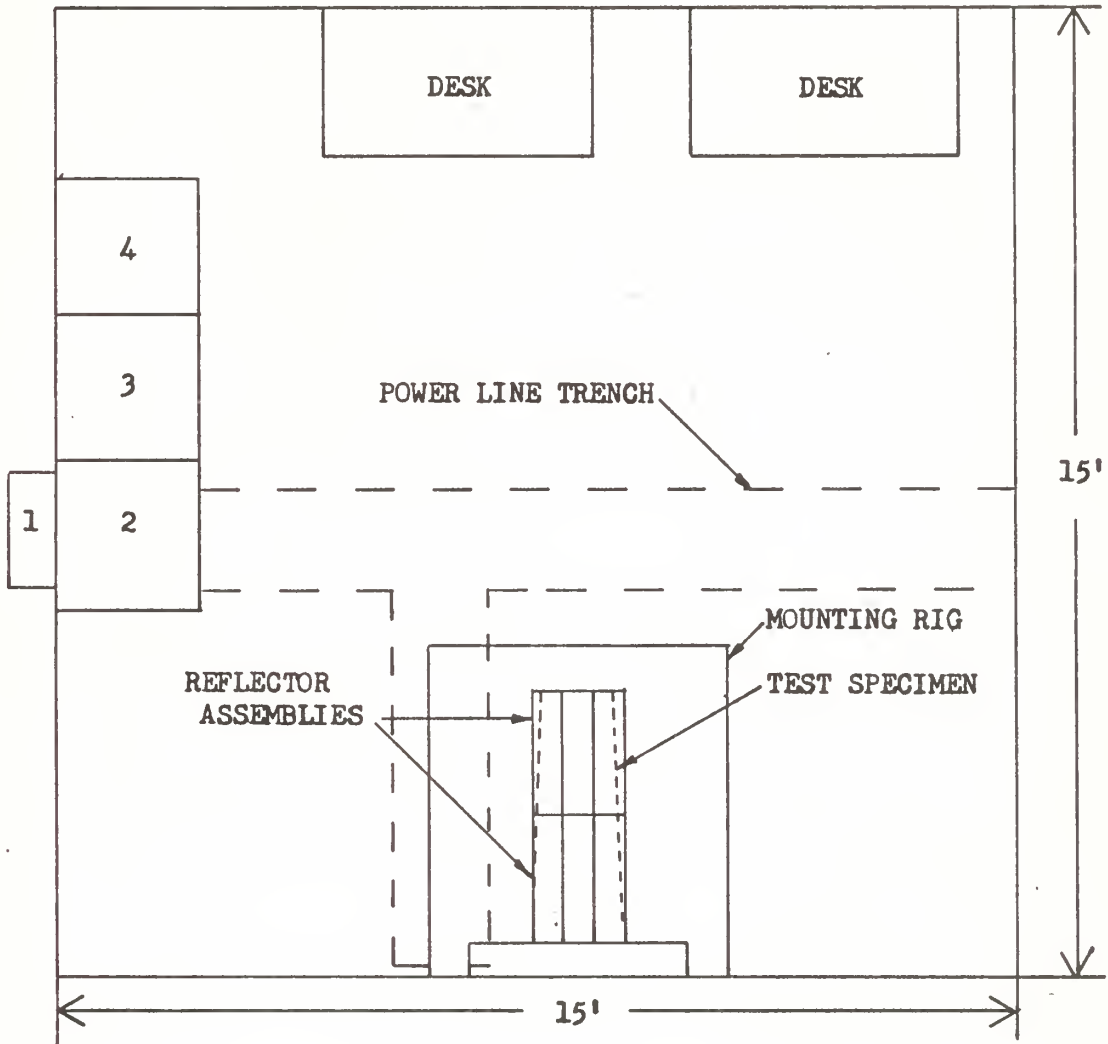
1. POWER INLET AND CIRCUIT BREAKER

2. POWER CONTROLLERS AND REGULATORS

3. RECORDERS

4. PROGRAMMER

programmer were placed next to the power regulators

so that one person may tend all controls. The mount-

ing platform with test specimen is close enough to

the operator positions to be observed, but not so close

as to be uncomfortable or dangerous at high tempera-

tures.

Alternate Proposals and Cost Analysis

In order to show the cost of building a facility

with a variety of capabilities, Table II has been

prepared. Proposals shown vary in cost from $20,780

to $11,500. All costs are approximate list prices

which may be discounted about 15%. This is approxi-

mately the cost for installation, however, so list

prices as shown can be considered realistic.

In selecting proposals, it was considered that

the test area of nine square feet should be kept con-

stant. This is considered a minimum test area for

maximum capability and flexibility of the testing

facility.

Proposal one is the one previously described. Pro-

posal two is identical except that no programmer is

included. This would restrict its use to applications

TABLE II COMPARISON OF SELECTED FACILITY PROPOSALS

PROPOSAL	1	2	3	4	5	6	7	8
CHANNELS	3	3	1	1	3	3	1	1
MAX TEMP	2000°F	2000°F	2000°F	2000°F	1000°F	1000°F	1000°F	700°F
TEST AREA	9 ft²	9 ft²	9 ft²	9 ft²	9 ft²	9 ft²	9 ft²	9 ft²
POWER (ALL 3 PHASE)	480 v 900 amp	480 v 900 amp	480 v 900 amp	480 v 900 amp	460 v 100 amp	460 v 100 amp	460 v 100 amp	460 v 100 amp
CONTROLLER REGULATOR	IGNITRON 1 $8,000	IGNITRON 1 $8,000	IGNITRON 1 $3,500	IGNITRON 1 $3,500	SCR $7,000	SCR $7,000	SCR $7,600 2	SCR $2,500
REFLECTORS & LAMPS	GOLD $1,280	GOLD $1,280	GOLD $1,280	GOLD $1,280	GOLD $1,280	GOLD $1,280	GOLD $1,280	ALUMINUM $1,000
RECORDERS	2-8 CHANNEL $6,000	2-8 CHANNEL $6,000	2-8 CHANNEL $6,000	2-8 CHANNEL $6,000	2-8 CHANNEL $6,000	2-8 CHANNEL $6,000	1-8 CHANNEL $3,000	2-8 CHANNEL $6,000
PROGRAMMER	3 CHANNEL $3,500	NONE 0	1 CHANNEL $2,000	NONE 0	3 CHANNEL $3,500	NONE 0	1 CHANNEL 0 2	NONE 0
THERMOCOUPLES & MISC.	$2,000	$2,000	$2,000	$2,000	$2,000	$2,000	$2,000	$2,000
TOTAL COST	$20,780	$17,280	$14,780	$12,780	$19,780	$16,280	$13,880	$11,500

1 NEEDS WATER COOLING

2 ONE UNIT INCLUDES CONTROLLER, REGULATOR, PROGRAMMER, AND ONE RECORDER

NOTE - ALL PRICES ARE APPROXIMATE AND INCLUDE INSTALLATION

where conditions under steady state temperatures (rather than transient temperatures) are being tested. As previously stated, a programmer could be added later with no further modifications to the facility necessary.

Proposals three and four are identical to one and two respectively except that only one channel of heating is provided. This would restrict applications to those of constant temperature. These facilities, therefore, would be unable to simulate the classical aerodynamic heating problem whereby a temperature gradient exists along the chordwise perimeter as shown in Fig. 2.

Although a one channel facility could later be converted to a multichannel facility, this is not as economical as initially purchasing the three channel unit.

Proposals five through seven are restricted in temperature capability to 1000^OF. Significant is the fact that current requirements are reduced to 100 amps at 460 volts for these facilities. This permits the less expensive, solid state SCR units to be used for power regulation. Other variations in these proposals

are as before except that in proposal seven, a single
unit is proposed which includes controller, regulator,
programmer, and recorder. An additional recorder
should still be purchased, however, if strain gage
information is to be recorded.

Proposal eight is the least expensive version pro-
posed. Temperature capability is restricted to 700°F
since uncooled aluminum reflectors are to be used.
Also, no programmer is included.

Table III shows a comparison of the eight proposals
with the cost of power extension and loading added to
obtain a total installation cost for any of the three
proposed locations. Note that the cost of power exten-
sion to location three for proposals five through eight
is significantly less than the others. The reason for
this is that 440 volt, 60 amp power already exists at
this location. The estimate of $1000 is therefore only
for a duplication of this power line to provide a total
of 120 amps.

A most significant conclusion which can be drawn
from this comparison is the fact that a 100% gain in
maximum temperature capability may be had for an average

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TABLE III TOTAL INSTALLED COST IN ALTERNATE LOCATIONS

PROPOSAL	1	2	3	4	5	6	7	8
FACILITY COST	$20,780	$17,280	$14,780	$12,780	$19,780	$16,280	$13,880	$11,500
POWER AND LOADING LOCATION 1	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
POWER AND LOADING LOCATION 2	6,000	6,000	6,000	6,000	3,000	3,000	3,000	3,000
POWER AND LOADING LOCATION 3	8,000	8,000	8,000	8,000	1,000	1,000	1,000	1,000
TOTAL COST LOCATION 1	21,780	18,280	15,780	13,780	20,780	17,280	14,880	12,500
TOTAL COST LOCATION 2	26,780	23,280	20,780	18,780	22,780	19,280	16,880	14,500
TOTAL COST LOCATION 3	28,780	25,280	22,780	20,780	20,780	17,280	14,280	12,500

of 20% increase in cost. Also, a 300% increase in temperature controllability (three channels versus one) may be had for an average 30% increase in cost. Proposal one is an excellent value, therefore, compared to the others and is recommended for installation at USNPS.

Clam Shell Furnance

Although the above described facility is flexible enough to handle most problems involving thermal effects on structures, there is one type of problem which it will not handle. That problem is one involving the use of cylindrical or near cylindrical shapes. An example might be an investigation of creep in a cylinder under compression in a universal testing machine and at a raised temperature. Recommended apparatus for this type of test is a clam shell furnace, again using quartz tube lamps as the source of heat. Such a furnace need not be capable of as rapid a response rate as a wing test facility. Since losses are low in an enclosed furnace and the nature of typical tests does not require rapid heating, the furnace need not require a great deal of power. Power regulation and control may therefore be

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accomplished by thyratron equipment presently owned by the Aeronautics Department. The only purchase necessary then to commence this type of testing is the furnace itself. The cost of this unit is $1100 complete with lamps.

Line Heater

One piece of equipment which would prove useful for laboratory work in conjunction with a thermoelasticity course is a line heater. This is a heating device which provides a high intensity heat source along a line from ten to thirty inches long. Such a device would prove useful in conjunction with an experiment involving stresses in a flat plate with a thermal gradient. Details of such a proposed experiment will be included in the following section.

The line heater recommended is of the infrared elliptical reflector type. This type would generate a highly concentrated radiant heat flux density along a thin line at the external focal axis of an elliptical reflector. The focal line is adjustable to accomodate various specimen thicknesses. Specimen temperature along the line can be raised to $2000^{O}F$ in less

than 30 seconds at 288 volts and 8.34 amps input in a
typical heater. Such a heater including the lamp sells
for about $150.

In order to regulate heater power to control speci-
men temperature, a power regulator unit will be needed.
For currents in this range solid state devices are
adequate and compact. The regulator should be capable
of manual control or accept an input signal from an
external automatic control system for maximum flexi-
bility. Such a power regulator is currently available
for about $200. The combined cost for this line heat-
ing facility therefore is about $350.

X. FACILITY USES

There are many ways in which a thermal effects
testing facility such as has been described may be
used. A primary use should be to provide facilities
for laboratory experiments in conjunction with a ther-
moelasticity course. Laboratory demonstration of
classic principles is mandatory for successful instruc-
tion in this field. In addition to this, students must
have a means of conducting experimental investigations
of high temperature effects in conjunction with thesis
work.

In this section, experiments and investigations
will be suggested which would fit into both programs.
Obviously, all possibilities for study will not be
discussed; however, it is hoped that ideas for inter-
esting and useful investigations will be precipitated
by the proposals which follow.

Determination of Thermal Stresses in a Flat Plate

An experiment which should be included as a lab-
oratory experiment in a thermoelasticity course is the
determination of stresses in a flat plate. The object
of this experiment is to compare experimentally deter-
mined stresses with calculated stresses in a flat plate

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of known dimensions and material and at an established temperature gradient.

Fig. 11 shows a top view of the proposed test setup. The flat plate is $\frac{1}{4}$ inch thick, 7075 aluminum alloy, measuring 16 inches by 24 inches. This will be mounted horizontally with ends restrained. The line heater is to be mounted under the plate and applies high intensity heat to the plate. On both edges a water pipe is mounted with the edge of the flat plate welded to the pipe. Eighteen strain gage rosettes and thermocouples are to be mounted as shown on the top of the plate. On both sides of the plate can be placed an asbestos cover.

With this setup, then, a linear temperature distribution can be established from the center outward with the outer edge temperature adjusted by varying the flow of cooling water through the pipes. The center temperature is adjusted by the power regulator. Edges will be restrained in a horizontal plane and either restrained or unrestrained in either longitudinal direction.

The procedure is to set 300°F at the center and 100°F at the edges by appropriate adjustment of the heater and water flow. When the temperature distribu-

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FIG. 11 TOP VIEW OF FLAT PLATE
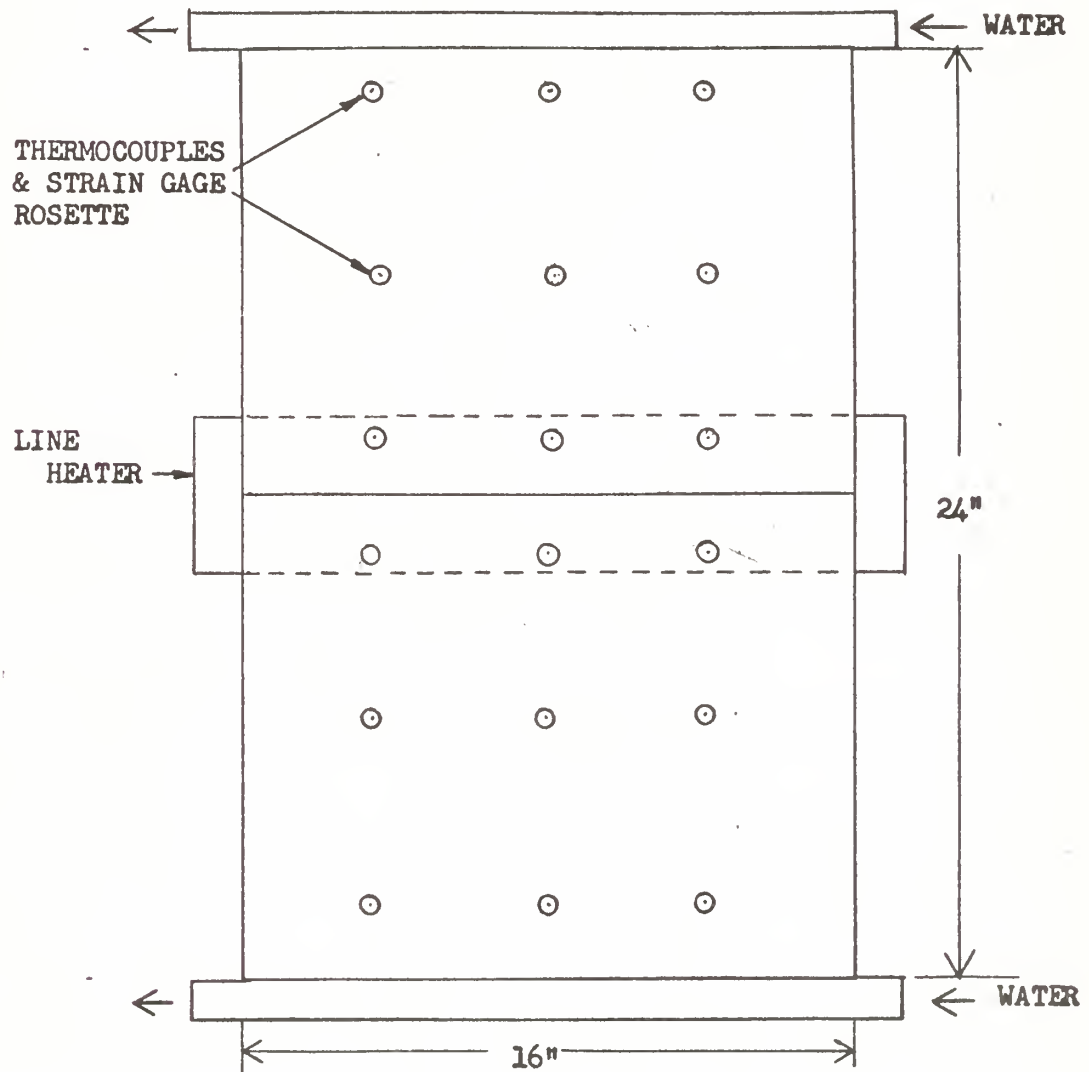
MATERIAL - 1/4" 7075 ALUMINUM ALLOY

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tion is observed to be linear from the center outward,
read all temperatures and strains. A second run with
the centerline at 500°F and the edge at 100°F should
also be performed. Each of these runs should be per-
formed with edges restrained in the longitudinal dir-
ection and then repeated with these edges unrestrained.

Several observations can be made when results are
plotted. First, the observed stresses can be compared
with previously calculated theoretical stresses. Second,
discrepancies in high and lower temperature runs can be
observed and attributed to changes in properties of the
metal. Third, the effects of end restraint can be ob-
served. Fourth, tendency of the plate to buckle will
be indicated by unsymmetrical strain readings in sym-
metrical locations.

Investigation of Thermal Stresses in a Missile Tail

To conduct this experiment, it would first be neces-
sary to obtain a tail surface from a supersonic missile.
Talos, Bomarc, and Tartar are designed to fly at about
Mach 3.0. Surfaces from these missiles would be of an
ideal size for testing in the proposed facility. Temp-
eratures encountered at Mach 3.0 range to about 600°F
at 50,000 ft.

The surface should be mounted horizontally in the testing facility. Eight strain gages and eight thermocouples should then be mounted on the test specimen and the reflector modules mounted so as to be one inch from the specimen surface.

With this setup, a series of tests can be run on the tail surface with the strain distribution measured for both constant temperatures and temperature gradients. First, the temperature should be raised in steps to $600^{O}F$. Next, the temperature should be raised at gradually increasing rates to $600^{O}F$ by use of the programmer. A plot of both strain and temperature versus time should be obtained for these runs on the multichannel recorders. Next, a flight profile can be programmed such that the leading and trailing edges are higher as the temperature increases to $600^{O}F$.

When these tests have been completed, the curves of temperature versus stress can be compared to theoretical calculations. Conclusions can then be drawn as to the effect of temperature, temperature gradients, and temperature rates of change on the tail surface. The type of construction can then be evaluated for use on high Mach number vehicles.

Equipment necessary for this experiment includes a testing facility capable of heating nine square feet of surface to 600°F. Three channel temperature control and programming will also be necessary. Proposals one or five as previously described would meet these requirements.

Investigation of Natural Frequency Change with Temperature of a Titanium Wing Model

A phenomona important in aeroelastic design is the change of natural frequency of an object with temperature change. In order to study this effect, it will first be necessary to construct a model. It is suggested that this be constructed of titanium. A six cell model such as shown in Fig. 12 would be adequate. Top and bottom are flat plates 1/8 inch thick. Webbing consists of seven channel sections of 0.062 inch sheet riveted to top and bottom. Such a model could readily be constructed in the USNPS machine shop. Titanium is recommended so that the model may be used for other tests in which material constants of this material are important.

When the model is constructed, mount it horizontally

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FIG. 12 MULTICELL WING MODEL
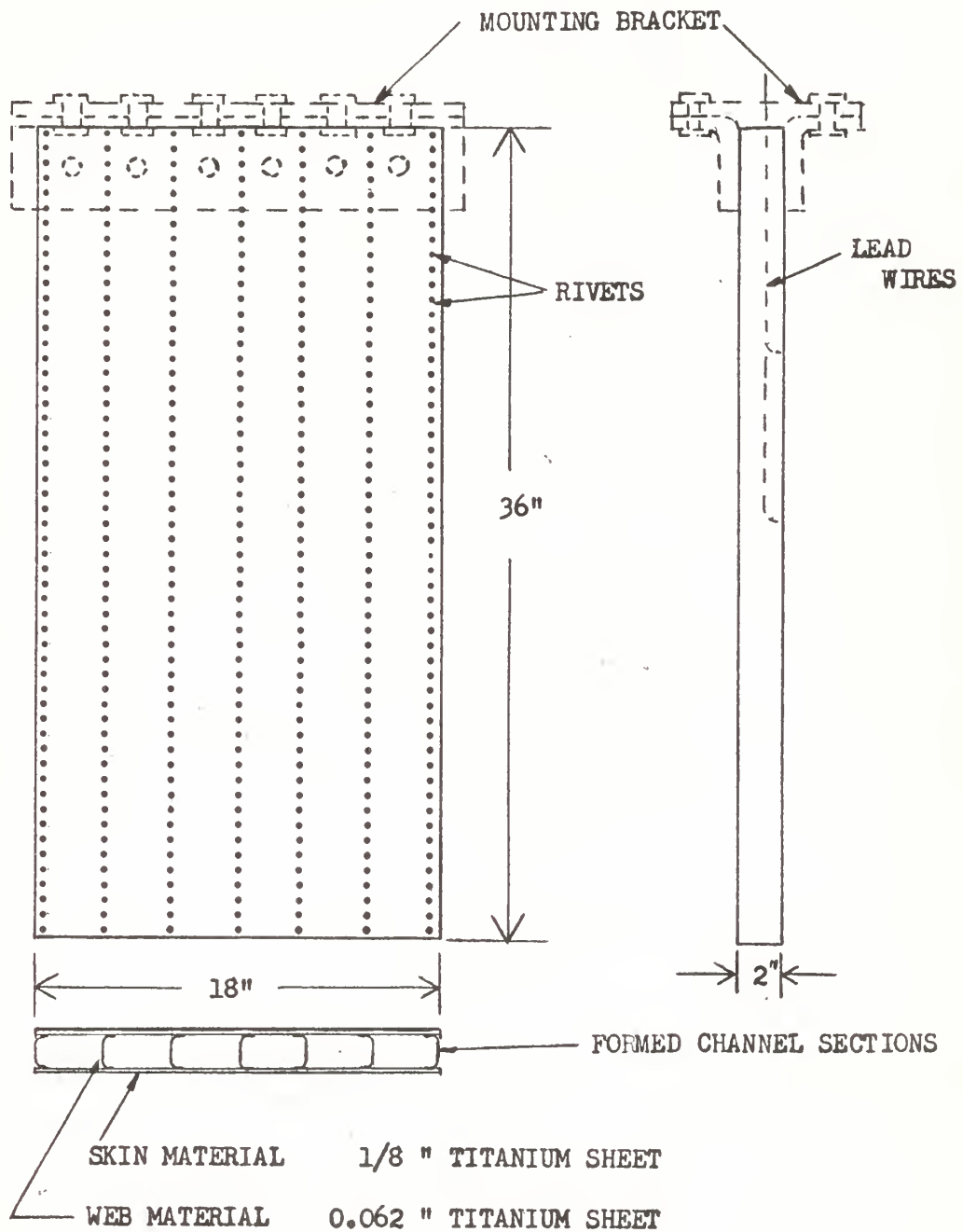
and attach a shaker to the root of the model. Attach

several high temperature strain gages near the free end

of the model. Determine the natural frequency by chang-

ing the frequency of the driving force and observing the

response on a cathode ray oscilliscope until a first

mode resonant peak is found. Then mount the heaters

above and below the model. Care must be taken that the

amplitude of vibration is not sufficient to strike the

quartz tube lamps. Next, the temperature of the model

should be raised in $100^{\circ}F$ increments and the natural

frequency determined at each step.

Results can be plotted in the form of a curve of

w_t/w_o versus temperature. It will be observed that the

natural frequency reduces with increasing temperature.

Observed results can then be compared with a theoreti-

cal analysis of the model.

Equipment necessary for this experiment is included

in proposals one, two, three, or four.

Investigation of Reradiation Effect on Heat Transfer in

a Multicell Wing

The principal mode of heat transfer in a multicell

wing at low temperature is conduction. Convection and

reradiation within the cells plays an insignificant roll. However, since heat transfer by radiation at a point inside the structure is proportional to the difference between the fourth power of the temperature at that point and the fourth power of all other points in the line of sight, the reradiation effect becomes increasingly important as the temperatures and the temperature gradients increase.

This experiment, therefore, is to evaluate the extent of reradiation on the temperature of the webs under various heat transfer conditions. The model described in the previous experiment will do as a test specimen.

The model should be mounted horizontally in the thermal testing facility with heaters on both sides. Thermocouples for temperature control should be attached to the surface and additional thermocouples attached along the centerline of the inner web for measurement. Then, from room temperature, the temperature of the model should be raised at a constant rate to a selected temperature which should be 100°F higher each run. Results will be in the form of a

plot of skin temperature and web temperature versus
time for each value of skin temperature selected.
Then the same procedure should be repeated, only this
time with a shielding installed along the inside of
the outside skin. This shielding should be designed
to reduce the effect of radiation of the inner skin
to a minimum.

Comparison of the result of the two sets of data
will show that reradiation causes the inner web to
rise in temperature faster than when shielded. Since
temperature gradients within the structure are the
primary cause of thermal stresses, it can be concluded
that, from this point of view, reradiation within the
structure is a desirable effect.

XI. CONCLUSIONS

In the analysis it was shown that the study of
thermal effects on aircraft structures is both neces-
sary and feasible. The most practical method of testing
is by use of an external radient heating source with
loads, when necessary, applied mechanically. Due to
the effects of scaling, the facility should be capable
of testing full scale components of large models.

The recommended facility is capable of heating nine
square feet up to 2000°F. Three channels are proposed
to simulate temperature gradients. The heat source is
an array of quartz tube lamps mounted in reflectors
with gold plating. Power regulation is of the ignitron
type. A three channel programmer and two eight channel
recorders are included. Control signal feedback is
surface temperature. Total cost including loading plat-
form and installation is $20,780. Alternate proposals
are included.

Also recommended for purchase is a clam shell heater
and a line heater. These cost $1100 and $350 respec-
tively including necessary support equipment. Some uses
for these items and the main testing facility have been
discussed in detail.

The total estimated cost for all recommended equipment is $22,230. Purchase of this equipment is considered a sound investment for USNPS.

XII.   BIBLIOGRAPHY

1. Horton, W. H., The Simulation of Heat and Load
    Environment in High Speed Vehicles, Engineers
    Thesis, Stanford University, Stanford California,
    1959.

2. Hoff, N. J., High Temperature Effects in Aircraft
    Structures, Pergamon Press, 1958.

3. Loria, J. C. and Blackstock, W. J., Aerodynamic
    Heating of Aircraft Structures in High-speed
    Flight, (Notes for a Special Summer Seminar),
    Chapter 18, Massachusetts Institute of Technology,
    1956.

4. O'Sullivan, W. J., Theory of Aircraft Structural
    Models Subject to Aerodynamic and External Loads,
    NACA TN 4115, Sept., 1957.

5. Molyneaux, W. G., A Consideration of the Similarity
    Requirements for Aerothermoelastic Tests on Reduced
    Scale Models, Proceedings of Symposium on Aerother-
    moelasticity, USAF ASD TR 61-645, Nov., 1961.

6. Ting, L,, Similarity Conditions for Testing High
    Speed Aircraft Models, Polytechnic Institute of
    Brooklyn, PIBAL Rep. no. 308, Nov., 1956.

7. Dugundji, J. and Calligeros, J., Similarity Laws for Aerothermoelastic Testing, Journal of Aerospace Sciences, Vol. 29, No. 8, August, 1962.

8. Loria, J. C., Mar, J. W., and Blackstock, W. J., The Design and Application of the MIT Radiant Heating Structural Test Facilities, Proceedings of the Society for Experimental Stress Analysis, Vol. XV, No. 1, 1957.

9. Trussell, D. H. and Weidman, D. J., A Radiant Heater to Simulate Aerodynamic Heating in a Wind Tunnel, NASA TN D-530, Nov. 1960.

10. Horton, W. H. and Johnson, R. W., The Stanford Aeronautics and Astronautics Department Hypersonic Wind Tunnel for Structural Research, SUDAER No. 164, August, 1963.

11. Campbell, J. D., A Feedback System for Automatic Control of Simulated Aerodynamic Heating, Engineers Thesis, Stanford University, Stanford, California, August, 1962.

12. Geronime, E. L., A Digital-Analog Computer Control for Use in an Aerodynamic Heat Simulation System, Engineers Thesis, Stanford University, Stanford, California, 1962.

13. Watson, R. M., A Heat Flow Measurement System for a Radiant Heating Test Facility, Engineers Thesis, Stanford University, Stanford, California, 1962.

14. Berman, J. S., Research into the Application of Radiant Heating Structural Testing of Aircraft at Elevated Temperatures, Final report submitted by Research Inc., Minneapolis, Minn. to WADC under contract AF 33(616)-2162, June 1954.

15. Horton, W. H., Experimental Methods in Kinetic Heat Tests, Aircraft Engineering, August, 1957.

16. Duberg, J. E., Some NACA Research on the Effect of Transient Heating on Aircraft Structures, Symposium on Structures for Thermal Flight, ASME, 1956.

17. Anonymous, Research Inc. (Controls Division) Catalogue of Thermal Testing Components, 1965.

18. Baird, R. B. and Brouns, R. C., Aircraft Structural Testing Techniques at Elevated Temperatures, Symposium on Structures for Thermal Flight, ASME, 1956.

19. Hoff, N. J. and others, Theory and Experiment in the Solution of Structural Problems of Supersonic Aircraft, Polytechnic Institute of Brooklyn, WADC TR 55-291, March, 1956.

20. Hoff, N. J. and others, Induction Heating and Theory in the Solution of Transient Problems of Aircraft Structures, Polytechnic Institute of Brooklyn, WADC TR 56-145, August, 1956.

21. Schultz, Experiments on Thermal Properties of Honey- comb Sandwiches Using a Radiant Heat Source, Thesis, USNPS, 1965.

22. Dove, R. C. and Adams, P. H., Experimental Stress Analysis and Motion Measurement, Clark E. Merrill Books, Inc., Columbus, Ohio, 1964.

APPENDIX A

Derivations of basic similarity parameters will be shown. The development is that of Dugundji (7). Symbols used are listed at the end of this Appendix.

Aerodynamic Similarity

Of primary importance in aerodynamic similarity are the pressure distribution and heat transfer rate due to external flow. The following are the applicable equations for a compressible, viscous, heat-conducting perfect gas:

Continuity:

$$\frac{\partial \rho}{\partial t} + \left(\frac{\partial}{\partial x_k}\right)(\rho\, v_k) = 0 \tag{1}$$

Momentum:

$$\rho\left[\frac{\partial v_j}{\partial t} + v_k \frac{\partial v_j}{\partial x_k}\right] = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_k}\left[\mu\left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j}\right)\right]$$
$$- \frac{2}{3}\frac{\partial}{\partial x_j}\left(\mu \frac{\partial v_k}{\partial x_k}\right) \tag{2}$$

Energy:

$$\rho\, C_p\left[\frac{\partial T}{\partial t} + v_k \frac{\partial T}{\partial x_k}\right] = \frac{\partial p}{\partial t} + v_k \frac{\partial p}{\partial x_k} + \frac{\partial}{\partial x_j}\left(k \frac{\partial T}{\partial x_j}\right) +$$
$$\mu\left[\frac{\partial v_j}{\partial x_k}\left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j}\right) - \frac{2}{3}\left(\frac{\partial v_k}{\partial x_k}\right)^2\right] \tag{3}$$

State:

$$P = \rho\left(\frac{R}{m}\right) T \tag{4}$$

The boundary conditions are:

Free stream:

$$\sqrt{v_k\, v_k} = V \qquad\qquad \rho = \rho_\infty \qquad\qquad T = T_\infty \tag{5}$$

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Body forces:

$$(x_j)_d = (x_j)_u + u_j \tag{6}$$

$$v_j = \frac{\partial u_j}{\partial t} \tag{7}$$

$$(T)_{AIR} = (T)_{BODY} \tag{8}$$

$$q_A = \left(-k \frac{\partial T}{\partial n}\right)_{gas} = \left(-k \frac{\partial T}{\partial n}\right)_{body} \tag{9}$$

If the following nondimentional quantities are intro-
duced:

$$\bar{x}_j = x_{j/L} \qquad\qquad \bar{\rho} = \rho/\rho_\infty \qquad\qquad \hat{c}_p = c_p/c_{p\infty}$$

$$\bar{t} = t/t_o \qquad\qquad \bar{P} = P/\rho_\infty v^2 \qquad\qquad \hat{k} = k/k_\infty \tag{10}$$

$$\bar{v}_j = v_{j/V} \qquad\qquad \bar{T} = T/T_o \qquad\qquad \hat{\mu} = \mu/\mu_\infty$$

$$\bar{U}_j = U_{j/U_o} \qquad\qquad\qquad\qquad \bar{K} = K/K_o$$

the applicable equations and boundary conditions reduce to:

$$\left(\frac{L}{Vt_o}\right) \frac{\partial \bar{\rho}}{\partial \bar{t}} + \frac{\partial}{\partial \bar{x}_k} (\bar{\rho}\,\bar{v}_k) = 0 \tag{11}$$

$$\bar{\rho}\left[\left(\frac{L}{Vt_o}\right) \frac{\partial \bar{v}_j}{\partial t} + \bar{v}_k \frac{\partial \bar{v}_j}{\partial \bar{x}_k}\right] = -\frac{\partial \bar{P}}{\partial \bar{x}_j} +$$

$$\left(\frac{\mu_\infty}{\rho_\infty VL}\right)\left[\frac{\partial}{\partial \bar{x}_k}\left(\hat{\mu} \frac{\partial \bar{v}_j}{\partial \bar{x}_k}\right) + \ldots\right] \tag{12}$$

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$$\bar{\rho}\,\hat{C}_p\left[\left(\tfrac{L}{Vt_0}\right)\tfrac{\partial\bar{T}}{\partial t} + \bar{v}_k\,\tfrac{\partial\bar{T}}{\partial\bar{x}_k}\right] = \left(\tfrac{V^2}{C_{P\infty}T_0}\right)\left[\left(\tfrac{L}{Vt_0}\right)\tfrac{\partial\bar{P}}{\partial t} + \bar{v}_k\,\tfrac{\partial\bar{P}}{\partial\bar{x}_k}\right]$$

$$+\left(\tfrac{k_\infty}{C_{P\infty}\mu_\infty}\right)\left(\tfrac{\mu_\infty}{\rho_\infty VL}\right)\tfrac{\partial}{\partial\bar{x}_k}\left(\hat{k}\,\tfrac{\partial\bar{T}}{\partial\bar{x}_k}\right) + \left(\tfrac{V^2}{C_{P\infty}T_0}\right)\left(\tfrac{\mu_\infty}{\rho_\infty VL}\right)$$

$$\left[\hat{\mu}\left(\tfrac{\partial\bar{v}_j}{\partial\bar{x}_k}\right)\left(\tfrac{\partial\bar{v}_j}{\partial\bar{x}_k} + \ldots\ldots\right)\right] \tag{13}$$

$$\bar{P} = \left[\tfrac{\mathcal{R}\,T_0}{(mV^2)}\right]\bar{\rho}\,\bar{T} \tag{14}$$

Free stream:

$$\sqrt{\bar{v}_k\,\bar{v}_k} = 1 \qquad \bar{\rho} = 1 \qquad \bar{T} = \tfrac{T_\infty}{T_0} \tag{15}$$

Body surface:

$$(\bar{x}_j)_d = (\bar{x}_j)_u + \left(\tfrac{u_0}{L}\right)\bar{u}_j \tag{16}$$

$$\bar{v}_j = \left(\tfrac{u_0}{L}\right)\left(\tfrac{L}{Vt_0}\right)\left(\tfrac{\partial\bar{u}_j}{\partial t}\right) \tag{17}$$

$$(\bar{T})_{air} = (\bar{T})_{body} \tag{18}$$

$$\left(\tfrac{k_\infty}{K_0}\right)\left(\hat{k}\,\tfrac{\partial\bar{T}}{\partial\bar{n}}\right)_{gas} = \left(\hat{K}\,\tfrac{\partial\bar{T}}{\partial\bar{n}}\right)_{body} \tag{19}$$

Upon consideration of the above equations it becomes apparent that if certain nondimensional parameters are identical for both the model and prototype, then the non-dimensional equations would be the same for both cases. This would lead to identical solutions for such unknowns as $\bar{P}$, $\bar{T}$, $\bar{\rho}$ and $\bar{v}_j$. Knowledge of these quantities would then lead to the solution for corresponding quantities

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on the prototype by use of equations (10). The so called "similarity parameters" that must be equal for both model and prototype are seen to be:

$$M_\infty \qquad Re_\infty \qquad Pr \qquad \gamma$$

$$\frac{Vt_o}{L} \qquad \frac{u_o}{L} \qquad \frac{T_o}{T_\infty} \qquad \frac{k_\infty}{K_o} \qquad\qquad (20)$$

$$\hat{C}_p \qquad \hat{k} \qquad \hat{\mu} \qquad \hat{K} \qquad \overline{T}_B \qquad \overline{u}_j$$

The first four parameters are Mach number, Reynolds number, Prandtl number and specific heat ratio for the free stream. The "unsteady parameter", $\frac{Vt_o}{L}$, "deflection parameter", $\frac{u_o}{L}$, and temperature ratio parameter, $\frac{T_o}{T_\infty}$, define appropriate reference time, t_o, reference deflection, u_o, and reference temperature, T_o, respectively. $\frac{k_\infty}{K_o}$ concerns similarity of body surface temperatures. Circumflexed quantities ($\hat{C}_p$, $\hat{k}$, $\hat{\mu}$, and $\hat{K}$) depend only on temperature and serve to require that variations of these quantities with temperature variations be the same for both model and prototype. This condition will be satisfied if the same gas and material is used in each case and if a common reference temperature, T_O, is used. $\overline{T}_B$ and $\overline{u}_j$ require body temperatures and deflections to be similar.

The above development, as stated, is for the general case of aerodynamic heating. If, however, it becomes

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convenient for a given problem to consider only the case of aerodynamic pressure distribution, appropriate simplifications can be made by assuming a nonviscous non-heat-conducting fluid. In this case the similarity parameters are reduced to:

$$M_\infty \ , \quad \gamma \ , \quad \frac{Vt_0}{L} \ , \quad \frac{u_0}{L} \ , \quad \bar{C}_p \ , \quad \bar{u}_j \qquad (21)$$

Other more specialized problems may be investigated by making the appropriate assumptions and thereby reducing the similarity parameters to the minimum number acceptable.

On the other hand it may be desirable to consider only the problem of aerodynamic heat transfer. In this case some additional considerations are necessary. A later development will show the necessity of the Biot - number parameter:

$$\frac{q_A}{K_0} \frac{L}{T_0} \qquad (22)$$

where q_A is the aerodynamic heat input rate. Since

$$q_A = \left(-k \ \partial T / \partial n \right)_{gas} \ ,$$

the Biot - number parameter reduces to the general similarity condition $\frac{k_\infty}{K_0}$ as shown in (9) and (19). This assumes of course that $\bar{T}$ is the same at corresponding points in the flow field. This will only occur if the complete set of dynamic similarity conditions (20) are satisfied.

A less restrictive example of aerodynamic heat trans-
fer can be examined. Consider the case of two-dimensional
steady flow over an isothermal, semi-infinite flat plate. The
Prandtl boundary layer equations yield:

For laminar flow:

$$q_A = 0.332 \left(\frac{k}{x}\right) \sqrt{R_e} \left(P_r\right)^{1/3} \left(T_{AW} - T_W\right) \tag{23}$$

For turbulent flow:

$$q_A = 0.029 \left(\frac{k}{x}\right) \left(R_e\right)^{0.8} \left(P_r\right)^{1/3} \left(T_{AW} - T_W\right) \tag{24}$$

where T_{AW} is the adiabadic wall temperature.

If the assumption is made that pressure gradients are
small, the properties in the above equations can be assumed
to be equal to those just outside of the boundary layer or
in the free stream. Using this assumption and placing the
above heating rates into the general Biot - number parameter
and combining with the general similarity parameters yields:

For laminar flow:

$$\left(\frac{k_\infty}{k_o}\right) \sqrt{R_{e_\infty}} \left(P_r\right)^{1/3}, \quad \frac{T_{AW}}{T_o}, \quad \overline{T_W} \hat{k}, \quad \hat{\mu} \tag{25}$$

For turbulent flow:

$$\left(\frac{k_\infty}{k_o}\right) \left(R_{e_\infty}\right)^{0.8} \left(P_r\right)^{1/3}, \quad \frac{T_{AW}}{T_o}, \quad \overline{T_W} \hat{k}, \quad \hat{\mu} \tag{26}$$

It can be concluded, then, that a temperature gradient along
the plate will not effect the similarity results. A small
conflict will result if laminar and turbulent flow are
present on a plate simultaneously. This will be small,
however. Of more importance would be insuring that the

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transition point occurs at the same relative position on both model and prototype. This could be forced with a trip wire if necessary.

Heat Conduction Similarity

The primary means of heat flow within a structure is conduction. The applicable formula is therefore the Fourier equation:

$$\frac{\partial}{\partial x_j}\left(k\,\frac{\partial T}{\partial x_j}\right) = \rho_B\,C\,\frac{\partial T}{\partial t} \tag{27}$$

the boundary conditions being:

at the surface

$$-k\left(\frac{\partial T}{\partial n}\right) = q_A - \epsilon_w\,\sigma\,T_w^{\,4} \tag{28}$$

and at $t = 0$, $T = T_{B_i}$ $\qquad\qquad\qquad$ (29)

The following nondimensional quantities are introduced:

$$\bar{x}_j = x_{j}/L \qquad\qquad \hat{k} = K/K_o$$

$$\bar{t} = t/t_o \qquad\qquad \hat{c} = C/C_o \tag{30}$$

$$\bar{T} = T/T_o \qquad\qquad \hat{\epsilon}_w = \epsilon_w/\epsilon_o$$

The following similarity parameters result:

$$\frac{q_A L}{K_o T_o}\,, \qquad \frac{\tilde{K}_o t_o}{L^2}\,, \qquad \frac{\epsilon_o\,\sigma\,T_o^{\,3}\,L}{K_o}$$

$$\hat{K}\,, \qquad \hat{c}\,, \qquad \hat{\epsilon}_w\,, \qquad T_{B_i}/T_o \tag{31}$$

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where thermal diffusivity $\tilde{K}_O = {}^{K_O}/_{\rho_B C_o}$

The first number is the Biot number, the second the

Fourier number defining the reference time t_o and the

third involves reradiation effects. Circumflexed para-

meters again require temperature variations of these

quantities to be similar for both model and prototype.

The last parameter defines reference temperature in

terms of initial body temperature.

As in the case of aerodynamic heating, less restric-

tive similarity conditions can be determined for certain

special cases by making appropriate assumptions. Consider

for example the case of heat conduction in a thin plate.

It is assumed that in a plate of thickness δ, tempera-

ture rises uniformly through the thickness (in the x_3

direction). The heat conduction equation becomes:

$$\frac{\partial}{\partial x_1}\left(K\delta\frac{\partial T}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(K\delta\frac{\partial T}{\partial x_2}\right) = \rho_B C \delta \frac{\partial T}{\partial t}$$
$$- q_A + \epsilon_w \sigma T_w^4 \tag{32}$$

Upon nondimensionalizing the above equation the following

similarity parameters evolve.

$$\frac{q_A L^2}{K_O T_O \delta} , \qquad \frac{\tilde{K}_O t_O}{L^2} , \qquad \frac{\epsilon_O \sigma T_O^3 L^2}{K_O \delta} \tag{33}$$

$$\hat{K} , \qquad \hat{C}, \qquad \hat{\epsilon}_w , \qquad T_{Bi}/T_O$$

These parameters are less restrictive than those in (31) since they allow the thickness ratio, δ/L, of the plate to be different in model and prototype. However, both must behave in a "thermally thin" manner as described by the assumption.

A further simplification can be made by assuming negligible heat conduction in the x_1 and x_2 directions as well as the x_3 direction. In this case the left side of equation (32) drops out and the similarity parameters become:

$$\frac{q_A\, t_o}{\rho_B c_o T_o \delta} \quad , \qquad \frac{\epsilon_o \sigma\, T_o^3 t_o}{\rho_B\, c_o\, \delta}$$

$$\hat{c} \quad , \qquad \hat{\epsilon}_w \quad , \qquad \frac{T_{Bi}}{T_o} \tag{34}$$

This would be appropriate for example in the simulation of heat conduction in a thin, solid, two-dimensional wing.

A built up structure could be considered using para-meters less restrictive than the general case (31) pro-vided certain simplifications could be made for the period of time involved. For example, consider a multiweb wing with a "thermally thin" skin. During the early heating period the assumption could be made that the skin heats up before any heat is conducted into the webs. This would permit use of expressions numbered (33). If, in addition,

it can be assumed that heat flow in the x_2 and x_3 directions is negligible, then the less restrictive parameters (34) could be used. This would be appropriate for only early time investigations; however, this is when large temperature differences between skin and web occur and, therefore, the greatest thermal stresses.

Stress and Deflection Similarity

The stress and deflection equations for a three--dimensional, isotropic, heated elastic body can be written as:

$$\frac{\partial \sigma_{jk}}{\partial x_k} = \rho_B \left(\frac{\partial^2 u_j}{\partial t^2} \right) - \rho_B g_i \tag{35}$$

$$\frac{1}{2} \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right) + \frac{1}{2} \left(\frac{\partial u_r}{\partial x_j} \frac{\partial u_r}{\partial x_k} \right) = \frac{1+\nu}{E} \sigma_{jk}$$

$$- \frac{\nu}{E} \delta_{jk} \sigma_{rr} + \delta_{jk} \alpha (T - T_{B_i}) \tag{36}$$

These represent nine equations with which to solve for six unknown stresses, σ_{ij}, and three unknown deflections, u_j . The boundary conditions are:

$$T = (T)_{Prescribed}$$

and either
$$u_j = (u_j)_{Prescribed}$$

or
$$\sigma_n = (\sigma_n)_{Prescribed} = P_A + P_F$$

$$\sigma_T = (\sigma_T)_{Prescribed} \approx 0$$

The following nondimensional quantities are introduced:

$$\bar{x}_j = x_j/L \qquad \widetilde{\sigma_{jk}} = \sigma_{jk}\big/\sigma_o \qquad \bar{E} = E/E_O$$

$$\bar{t} = t\big/t_o \qquad \bar{U}_j = U_j\big/U_O \qquad \hat{\alpha} = \alpha\big/\alpha_o \qquad (37)$$

$$\bar{T} = T/T_O$$

Following procedures as before yields the similarity

parameters for the general case:

$$\frac{\sigma_o L}{E_o\, \bar{u}_o}\;, \qquad \frac{\alpha_o T_o L}{u_o}\;, \qquad \frac{u_o}{L}\;, \qquad \frac{P_A}{\sigma_o}\;, \qquad \frac{P_E}{\sigma_o}$$

$$\frac{\rho_B\, u_o L}{\sigma_o\, t^2}\;, \qquad \frac{\rho_B\, g L}{\sigma_o}\;, \qquad \nu\;, \quad \bar{E}\;, \quad \hat{\alpha}\;, \quad \bar{T} \qquad (38)$$

   The first parameter is the basic stress parameter.

The second has to do with thermal stresses. The third

parameter serves to define the reference deflection $, u_o \,$

along with other parameters. It may be dropped for small

deflections. The next two parameters define the reference

stress $, \sigma_o$. The first parameter in the bottom line concerns

dynamic vibrations and defines reference time. The next

parameter has to do with gravitational loadings. Meanings

of remaining parameters are self-evident.

   As in previous considerations, a set of less restric-

tive parameters can be obtained for specialized cases

involving stress and deflection similarity. One structure

important to the aeronautical engineer is the plate.
Wing panels are plates and sometimes entire wings can
be treated as plates provided they are thin and low
aspect ratio.

Appropriate for the case of large-deflection of a
heated and laterally loaded isotropic plate of variable
stiffness are the von Karman equations:

$$\frac{\partial^2}{\partial x_1^2}\left[D\left(\frac{\partial^2 u_3}{\partial x_1^2} + \nu\,\frac{\partial^2 u_3}{\partial x_2^2}\right)\right] + \frac{\partial^2}{\partial x_2^2}\left[D\left(\frac{\partial^2 u_3}{\partial x_1^2} + \nu\,\frac{\partial^2 u_3}{\partial x_1^2}\right)\right] +$$

$$2(1-\nu)\,\frac{\partial^2}{\partial x_1 \partial x_2}\left[D\,\frac{\partial^2 u_3}{\partial x_1 \partial x_2}\right] = \Delta p + \frac{\partial^2 F}{\partial x_2}\,\frac{\partial^2 u_3}{\partial x_1^2} \qquad (40)$$

$$+ \frac{\partial^2 F}{\partial x_1^2}\,\frac{\partial^2 u_3}{\partial x_2^2} - 2\,\frac{\partial^2 F}{\partial x_1 \partial x_2}\,\frac{\partial^2 u_3}{\partial x_1 \partial x_2} - \frac{1}{1-\nu}\left(\frac{\partial^2 M_T}{\partial x_1^2}\right.$$

$$\left. + \frac{\partial^2 M_T}{\partial x_2^2}\right)$$

$$\frac{\partial^2}{\partial x_1^2}\left[\frac{1}{B}\left(\frac{\partial^2 F}{\partial x_1^2} - \nu\,\frac{\partial^2 F}{\partial x_2^2}\right)\right] + \frac{\partial^2}{\partial x_2^2}\left[\frac{1}{B}\left(\frac{\partial^2 F}{\partial x_2^2} - \nu\,\frac{\partial^2 F}{\partial x_1^2}\right)\right] + \qquad (41)$$

$$2(1+\nu)\,\frac{\partial^2}{\partial x_1^2 \partial x_2^2}\left[\frac{1}{B}\,\frac{\partial^2 F}{\partial x_1 \partial x_2}\right] = (1-\nu^2)\left[\left(\frac{\partial^2 u_3}{\partial x_1 \partial x_2}\right)^2\right.$$

$$\left. - \frac{\partial^2 u_3}{\partial x_1^2}\,\frac{\partial^2 u_3}{\partial x_2^2}\right] - \frac{\partial^2}{\partial x_1^2}\left[\frac{N_T}{B}\right] - \frac{\partial^2}{\partial x_2^2}\left[\frac{N_T}{B}\right]$$

where: $\Delta p(x_1, x_2,\ t)$ is lateral loading on the plate due
to aerodynamic, inertial, gravitational of other applied
forces.

$$\Delta p = \Delta p_A - m\left(\frac{\partial^2 u_3}{\partial t^2}\right) - mg + P_F \qquad (42)$$

The stress function F is defined as:

$$\frac{\partial^2 F}{\partial x_1^2} = N_{22} = \int_{-S/2}^{S/2}\sigma_{22}\,dx_3, \qquad \frac{\partial^2 F}{\partial x_2^2} = N_{11} = \int_{-S/2}^{S/2}\sigma_{11}\,dx_3, \qquad -\frac{\partial^2 F}{\partial x_1 \partial x_2} = N_{12} = \int_{-S/2}^{S/2}\sigma_{12}\,dx_3 \quad (43)$$

The extensional stiffness B is defined as:

$$B = \int_{-\delta/2}^{\delta/2} \frac{E}{1-\nu^2} \, dx_3 \qquad (44)$$

The bending stress D is defined as:

$$D = \int_{-\delta/2}^{\delta/2} \frac{E x_3^2}{1-\nu^2} \, dx_3 \qquad (45)$$

The thermal force N_T is defined as:

$$N_T = \int_{-\delta/2}^{\delta/2} E\alpha \left[T - T_{B_i} \right] dx_3 \qquad (46)$$

The thermal moment M_T is defined as:

$$M_T = \int_{-\delta/2}^{\delta/2} E\alpha \left[T - T_{B_i} \right] x_3 \, dx_3 \qquad (47)$$

Nondimensional quantities to be introduced are:

$$\bar{x}_i = x_i / L \qquad \bar{F} = F/N_0 L^2 \qquad \widehat{N_T} = N_T/N_{T_0}$$

$$\bar{u}_3 = u_3/u_0 \qquad \hat{B} = B/B_0 \qquad \widehat{M_T} = M_T/M_{T_0} \qquad (48)$$

$$\overline{N_{ij}} = N_{ij}/N_0 \qquad \hat{D} = D/D_0 \qquad \overline{\sigma_{ij}} = \sigma_{ij}/\sigma_0$$

$$m = m/m_0 \qquad \overline{\Delta P_A} = \Delta P_A/(\Delta P_A)_0$$

The resulting similarity parameters which occur when

the equations are made nondimensional are:

$$\frac{\Delta P_A L^4}{D_0 u_0} \qquad \frac{m_0 L^4}{D_0 t_0^2} \qquad \frac{m_0 g L^4}{D_0 u_0} \qquad \frac{P_F L^4}{D_0 u_0}$$

$$\frac{N_0 L^2}{D_0} \qquad \frac{L^2 M_{T_0}}{D_0 u_0} \qquad \frac{B_0 u_0^2}{N_0 L^2} \qquad \frac{N_{T_0}}{N_0} \qquad (49)$$

$$\hat{B} \qquad \hat{D} \qquad \widehat{N_T} \qquad \widehat{M_T} \qquad \overline{m} \qquad \overline{\Delta P_A}$$

If deflections are small, $\frac{B_0 u_0^2}{N_0 L^2}$ may be neglected.

These parameters may be reduced by considering a

plate of thickness $\delta = \tau L \bar{\delta}$. Thus the quantities B_0,

D_0, N_{T_0}, M_{T_0}, and N_0 can be related to δ using equations

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(44) through (47) and the parameters become after rearrange-

ment:
$$\frac{(\Delta p_A)_o\, L}{E_o\, \uparrow^3\, u_o} \; , \qquad \frac{\rho_B\, L^2}{E_o\, \uparrow^2\, t_o^2} \; , \qquad \frac{\rho_B\, g\, L^2}{E_o\, \uparrow^2\, u_o}$$

$$\frac{p_F\, L}{E_o\, \uparrow^3\, u_o} \; , \qquad \frac{\sigma_o}{E_o\, \uparrow^2} \; , \qquad \frac{\alpha_o\, T_o\, L}{u_o\, \uparrow} \; , \qquad \frac{u_o}{L\, \uparrow} \qquad (50)$$

$$\frac{\alpha_o\, T_o}{\uparrow^2} \; , \qquad \nu \; , \qquad \overline{T} \; , \qquad \overline{\delta} \; , \qquad \overline{\Delta p_A}$$

Again $\frac{u_o}{L\uparrow}$ may be neglected for small deflections, however,

$\frac{\alpha_o\, T_o\, L}{u_o\, \uparrow}$ and $\frac{\alpha_o\, T_o}{\uparrow^2}$ must be mutually satisfied. Note that

these parameters are less restrictive than the general

plate parameters (49) since they permit variation of the

thickness ratio $\uparrow$ of the plates.

Use of the general plate parameters (49) has been

indicated to provide similarity between multiweb and multi-

spar wings of differing internal arrangement. Such "equi-

valent plate" wings have been discussed by Ting and others.

(4, 5, 6)

Complete and Incomplete Aerothermoelastic Testing

Complete aerothermoelastic testing is achieved by

the placing of a model in a wind tunnel such that the air

stream will simulate completely the aerodynamic-heating-

-rate environment required for the model. In order for

this to be accomplished, the general similarity parameters

of the three previous sections must be combined. This was

done and after some rearrangement the following general

similarity parameters resulted:

$$M_\infty \ , \qquad Re_\infty \ , \qquad \frac{k_\infty}{K_o} \ , \qquad \frac{\rho V^2}{E_o} \ , \qquad \alpha_o T_o \ , \qquad \frac{\rho_B}{\rho_\infty}$$

$$\frac{T_o}{T_\infty} \ , \qquad \frac{\tilde{K}_o t_o}{L^2} \ , \qquad \frac{V t_o}{L} \ , \qquad \frac{u_o}{L} \ , \qquad \frac{\sigma_o}{E_o} \ , \ \frac{P_F}{E_o}$$

$$\frac{\epsilon_o \sigma T_o^3 L}{K_o} \ , \qquad \frac{\rho_B g L}{\rho_\infty V^2} \ , \qquad P_r \ , \ \sigma \ , \ \nu \ , \qquad \hat{k} \ , \ \hat{c}_p$$

$$\hat{\mu} \ , \qquad \hat{K} \ , \qquad \hat{c} \ , \qquad \hat{E} \ , \ \hat{\alpha} \ , \qquad \hat{\epsilon}_w \ , \ \frac{T_{Bi}}{T_o}$$

(51)

Difficulties connected with the reconciliation of the

first five parameters have been found to be insurmount-

able for any but a scale factor of unity. Attempts have

been made to overcome this problem by the use of gasses

other than air and materials other than those used on the

prototype for tests. Even then results are generally

poor. For these reasons attempts at complete aerother-

moelastic testing are not recommended at USNPS.

If complete aerothermoelastic testing is not feasible,

a testing technique called "incomplete aerothermoelastic

testing" is available. In this case means other than air

flow alone provide net aerodynamic pressure (P_{net}) and net

heat transfer input (q_{net}) to the model. Parameters appro-

priate to this case were obtained by combining parameters

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(31) and (39). After rearrangement the following para-

meters result:

$$\frac{q_{net}\ L}{K_o\ (T_o-T_{Bi})}\ ,\qquad \frac{p_{net}}{E_o}\ ,\qquad \alpha_o(T_o-T_{Bi})$$

$$\frac{\rho_B L^2}{E_o t_o^2}\ ,\qquad \frac{\tilde{K}_o t_o}{L^2}\ ,\qquad \frac{u_o}{L}\ ,\qquad \frac{\sigma_o}{E_o}\ ,\qquad \frac{\rho_B g L}{\sigma_o}\qquad (52)$$

$$\hat{\nu}\ ,\qquad \hat{K},\qquad \hat{C},\qquad \hat{E},\qquad \hat{\alpha}$$

In the evaluation of p_{net} and q_{net} an estimate must first

be made of the expected aerodynamic pressure p_A, aerody-

namic heating rate q_A, and radiation loss $\epsilon \sigma T_w^4$ for

both prototype and model. Then the necessary p_F and q_F is

added externally to the model to satisfy the parameters.

In spite of the apparent crudeness of this method, it is

commonly used for structural testing at high temperature.

A further degree of freedom may be attained if it

can be assumed that deflections are small for a given

problem. In this case the $\frac{u_o}{L}$ parameter need not be satis-

fied and the second, third, and seventh parameters can be

expressed as:

$$\frac{p_{net}\ L}{E_o\ u_o}\ ,\qquad \frac{\alpha_o(T_o - T_{Bi})L}{u_o}\ ,\qquad \frac{\sigma_o L}{E_o\ u_o}\qquad (53)$$

These are less restrictive since they allow greater free-

dom in that u_o can be chosen arbitrarily for both proto-

type and model.

If problems associated with adding P_F forces to the
model are great, it may be desirable to simulate the com-
plete p_{net} force by the air stream. In this case:

$$p_{net} = P_A = \rho_\infty V^2 \, \bar{p}$$

where $\bar{p}$ is established by use of the general parameters
(21). The similarity parameters become:

$$\frac{q_{net} \, L}{K_o \, (T_o - T_{Bi})} \quad , \quad M_\infty \, , \quad \frac{\rho_\infty V^2}{E_o} \quad , \quad \alpha_o(T_o - T_{Bi})$$

$$\frac{\rho_B}{\rho_\infty} \quad , \quad \frac{\tilde{K}_o t_o}{L^2} \quad , \quad \frac{V t_o}{L} \quad , \quad \frac{u_o}{L} \quad , \quad \frac{\sigma_o}{E_o} \quad , \quad \frac{\rho_B g L}{\rho_\infty V^2} \tag{54}$$

$$\gamma \quad , \quad \nu \quad , \quad \hat{c}_p, \quad \hat{K}, \quad \hat{c}, \quad \hat{E}, \quad \hat{\alpha}$$

To satisfy the first parameter it will be necessary to
add q_F to the model, possibly by use of radiant heaters
within the wind tunnel. Tests on a device of this type
have been completed by NASA (9).

TABLE OF SYMBOLS

b   = semichord of wing

B   = plate extensional stiffness

C   = specific heat of body material

c_p = specific heat of gas at constant pressure

D   = plate bending stiffness

E   = Young's modulus

F   = stress function

G   = shear modulus

h   = wing thickness

g   = acceleration due to gravity

I   = area moment of inertia

J   = torsional stiffness constant

K   = heat conductivity of body material

k   = heat conductivity of gas

$\tilde{k}$ = thermal diffusivity

L   = characteristic length

M   = Mach number

m   = molecular weight, mass

M_t = plate midplane thermal moment per unit length

N_t = plate midplane thermal force per unit length

n   = normal direction

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p   = pressure

Pr = Prandtl number

q   = heat flux per unit area

$\mathcal{R}$ = universal gas constant

R   = resistance

Re = Reynolds number

T   = temperature

T_l = temperature distribution

T_{aw} = adiabadic wall temperature

t   = time

u_j = displacement component, deflection component

V   = free stream velocity, voltage

v_j = velocity component, deflection component

x_j = rectangular coordinate

α = coefficient of thermal expansion

γ = ratio of specific heats

δ = thickness

ϵ = emissivity

ϵ_{jk} = strain component

μ = viscosity of gas

ν = Poisson's ratio

ρ = density

σ_{jk} = stress component

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σ_n, σ_t = normal and tangential stresses on boundary

$\uparrow$ = thickness ratio

<div align="center">SUBSCRIPTS</div>

a = of aerodynamic origin

aw = adiabadic wall

b = body

d = deformed state

f = of nonaerodynamic origin

i = initial value

j, k, r = summation indices 1, 2, 3

m = model

o = reference value

p = prototype

u = undeformed state

w = wall value

∞ = free stream value
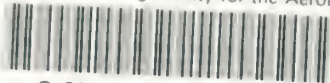
<div align="center">SUPERSCRIPTS</div>

$(\bar{\ })$ = nondimensional quantity

$(\hat{\ })$ = nondimensional, temperature-dependent property variation

Note: Tensor summation convention for repeated indices is used.